A market structure for an environment with heterogeneous job-matches, indivisible labor and persistent unemployment

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Abstract

The Mortensen–Pissarides model with heterogeneous job-matches and persistent unemployment has become a popular workhorse for studying aggregate phenomena in the labor market. This paper develops a microeconomic structure decentralizing a generalized social planner’s version of this model. It is shown in the presence of the indivisibility of labor, heterogeneous job-matches, and persistent unemployment, that different households can fully insure themselves against variations in wealth, if they have access to a complete insurance market. Also factor prices and matching rates are derived.

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Exactly because their (Kydland and Prescott’s) model carries predictions for so wide a range of evidence, it has been subjected to an unusually wide range of empirically-based criticism: here is a macroeconomic model that actually makes contact with microeconomic studies in labor economics! The chances that the model will survive this criticism unscathed are negligible, but this seems to me exactly what explicit theory is for, to lay bare the assumptions about

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behavior on which the model rests, to bring evidence to bear on these assumptions, to revise them when needed, and so on.

(Robert E. Lucas Jr., 1987, 47)

1. Introduction

In a recent series of papers, Mortensen and Pissarides (1993, 1994) have developed an analytical framework that has become widely known as the Mortensen-Pissarides model, or the MP-model in brief. Their analysis builds upon earlier work by Pissarides (1985) in which he introduces productivity differences across firms. The authors combine this element with a dynamic stochastic version of the trade frictions approach to unemployment originally introduced by Pissarides (1985). This approach uses elements of two-sided search for studying aggregate phenomena in the labor market, and introduces unemployment as a persistent phenomenon. Two distinctive features of this approach are search externalities that act as the main propagation mechanism of exogenous shocks and its explanation of wage determination. Search externalities arise, as the rate of job contacts made by searching workers and firms depends on the number of traders present on both sides of the market. Mortensen and Pissarides model the presence of such externalities as well as the process of creating a match by the artifact of a matching technology that links matching output (the flow of newly formed job-worker pairs) to matching input (searching workers and job-advertising firms). They work with various wage determination schemes. Their analytical framework is path-breaking as it enables the authors to study firms' endogenous decisions to create or destroy a job-match in a dynamic general equilibrium setting. It has generated considerable attention in the profession. It has been extended and modified in various directions and has proven to be a valuable workhorse for studying aggregate labor market phenomena.¹

In this paper, I provide a microeconomic structure that can be used to reformulate the social planner's problem of a generalized version of the MP-framework as a decentralized market economy. While this microeconomic structure lends itself to an interpretation that establishes its relationship to the existing literature on trade frictions in the labor market, it is by no means unique. The decentralization scheme provided is just one out of the set of many other feasible characteriza-

¹ Mortensen (1994) uses the model to analyze the cyclical behavior of job and worker flows. Garibaldi (1994) does the same when severance payments are present. Similarly, Millard and Mortensen (1994), and Cabrales and Hopenhayn (1995) use the model to study the welfare implications of a set of labor-related taxes and subsidies. Merz (1994) synthesizes the original MP-model and the one-sector stochastic growth model as formulated by Kydland and Prescott (1992), and Long and Plosser (1983). This enables her to introduce physical capital and standard functional forms of utility and production technology into the MP-framework, thereby rendering its predictions comparable to those of existing models of the business cycle.
tions. The model presented deviates from the MP-framework in several respects. It is more general, as it allows for risk aversion among workers, a more broadly defined production function, and physical capital. But it is more restrictive, as it focuses on the special case of a Pareto optimal environment.

I spell out the necessary assumptions to decentralize the social planner's problem of the generalized version of the MP-framework as a market economy. Knowing the exact elements required for constructing a benchmark case for the Pareto optimal version of the MP-model facilitates the relaxation of the Pareto optimal environment through the modification of my assumptions.

The MP-model exhibits indivisible labor and persistent unemployment which arises from the trade frictions in the labor market. Households have access to a complete insurance market before any other economic activity takes place. Furthermore, their preferences are separable in consumption and leisure. The assumptions imply that households can fully insure themselves against differences in wealth arising from different labor market histories. This extends the result by Rogerson (1988) and Hansen (1985) which they derive for an environment with indivisible labor and non-persistent unemployment in the MP-model. In the Hansen–Rogerson economy, an employment lottery is introduced to determine on a period-to-period basis the number of households to be employed and unemployed, respectively, and to fully insure households against the risk of becoming unemployed. I also assume the existence of the equilibrium factor prices and matching rates for firms' job-vacancies required to show the equivalence between the planning version and the market version of my model economy.

The rest of this paper is organized as follows. I start out by presenting the social planner's version of a generalized Mortensen–Pissarides economy, and go on to develop a corresponding market structure. I show that when households have access to complete insurance markets before any other economic activity takes place, they can fully insure themselves against income variations due to unemployment. I also derive the factor prices and matching rates for firms' job-vacancies that establish an equivalence between the decentralized market version and the generalized social planner's version of the Mortensen–Pissarides economy. The final section presents conclusions.

2. The economic environment

The stylized economy is populated by a continuum of infinitely-lived worker-households with names on the closed interval [0,1] and a continuum of identical firms. Households and firms exchange goods and factors of production. While goods and the factor capital are exchanged in perfectly competitive markets, labor is traded in a process that exhibits search externalities. Search externalities arise, since trade frictions characterize the process in which households and firms exchange labor services. The rate at which unemployed workers and firms make
job-contacts depends on the tightness of the labor market, i.e., the relative number of traders present on both sides of the market. For any given trader, a positive externality arises whenever the number of traders on the opposite side of the market increases, and vice versa. This situation is referred to as a thin-market externality, since it facilitates trade. Alternatively, an increase in the number of traders on the same side of the market renders trade more difficult. This situation is commonly referred to as congestion. The events in the labor market are sketched in Fig. A.1 of the appendix. This figure depicts various states in which worker–households can be found and the various decisions and exogenous forces that make them move across the states.

All households are members of the labor force, \( L \), that is assumed to be constant and normalized to one, as there is no labor-leisure choice in the model. It is depicted as a circle. This assumption is based on the empirical observation that movements into and movements out of the labor force are smaller than those within the labor force. Worker–households differ with respect to their status in the labor force. At any point in time, workers can be engaged in a job-match and employed, \( E \), or they can be participants of the unemployment pool, \( U \). Each match is assumed to face a constant per-period probability, \( \delta_N \), of being dissolved with the worker becoming unemployed. This assumption captures the flow of workers out of employment which occurs independently from any firm’s decision such as retirements, for example.

Even though all workers are identical ex ante, once they are employed, their job-match is exposed to recurring aggregate productivity shocks, and also to idiosyncratic shocks which may change their idiosyncratic labor productivity. Ex post heterogeneity exists among job-matches, and firms endogenously decide which job-match to dissolve and which to maintain. When a job-match is dissolved, a dismissed worker can become unemployed for two separate reasons. Either the worker is dismissed because an idiosyncratic shock decreases his job-match's productivity below the firm’s reservation productivity, \( R_i \), or because the reservation productivity increases beyond his match’s given labor productivity. Of course, a job-match can be dissolved if both events occur simultaneously.

If unemployed, a worker faces a certain chance of being rematched with another firm, changing the worker’s status from unemployed to employed. This probability varies with the tightness of the labor market measured by the total number of vacancies, \( V_t \), relative to the total number of unemployed workers. It increases with an increase in the market thickness brought about by a relative increase in the number of listed job-vacancies. It decreases with congestion caused by a relative increase in unemployment. Creating new job-matches is expensive, as firms need to post vacancies in order to recruit applicants from the pool of unemployed.

I first present the social planner’s version of my model economy and then develop an equivalent market economy.
2.1. The social planning problem

All new job-matches in the model economy are equally productive. Heterogeneity in labor productivity is introduced via the assumption that matches are subsequently exposed to idiosyncratic productivity shocks orthogonal to a common aggregate technology shock. The aggregate technology shock $z_t$ is assumed to follow an AR(1) process with the following law of motion:

$$z_{t+1} = \rho z_t + \epsilon_{t+1},$$

where $0 < \rho < 1$, and $\epsilon_t$ is an i.i.d. random variable drawn from a normal distribution with mean zero and standard deviation $\sigma_e$. Shocks to the idiosyncratic productivity level $x_t$ follow a stochastic process that is uniformly distributed across the interval $[-\gamma, \gamma]$:

$$x_{t+1} = \begin{cases} x_t & \text{with probability } (1 - \zeta), \\ x \sim G : [-\gamma, \gamma] \rightarrow [0, 1] & \text{with probability } \zeta g_x, \end{cases}$$

where $g_x = G'(x)$. This process implies the following probability density function, $g_x$, and cumulative distribution function, $G(x)$, for the random variable $x$:

$$g_x = \frac{1}{2\gamma}, \quad G(x) = \frac{x + \gamma}{2\gamma} \quad \forall x \in [-\gamma, \gamma].$$

Hence, $x_t$ is independently and identically distributed across job-matches, first-order Markov, and positively correlated with bounded support. With probability $(1 - \zeta)$ a job-match is characterized by the same idiosyncratic productivity in two consecutive periods. With probability $\zeta$ this productivity level will change. In that case, the new level will be independent of what it was initially, i.e., draws from the distribution $G$ are i.i.d. Furthermore, it is assumed that all new job-matches exhibit the highest possible level of idiosyncratic productivity, $\gamma$. This assumption captures the idea that newly created jobs are more productive than existing ones, since they can be located in either physical, technology, or commodity space and thereby reflect current information about future profitability. Technically, this assumption helps avoid introducing the distribution of new job-matches across idiosyncratic productivity levels as another variable in the state space of the optimization problem, thereby keeping the problem tractable. It also introduces $\gamma$ as a mass point which makes Lebesgue integration – rather than Riemann integration – necessary when computing total employment.

The total labor force in the economy is assumed to be constant over time and is normalized to one. Total time endowment is also normalized to one with agents being able to divide up their available time between hours spent working, and enjoying leisure. The decision making of consumer-workers and firms in general equilibrium can be summarized by the outcome of the following representative agent's welfare maximization problem. The agent orders time paths
of consumption services, $C_t$, and total hours spent working according to the criterion

$$E_z \sum_{t=1}^{\infty} \beta^t U(C_t, H_t),$$

(3)

where $0 < \beta < 1$ denotes the common discount factor and $E_z$ represents the expectation operator that takes expectations with respect to the random variable $z$. Preferences are specified as additively separable between consumption and hours spent working:

$$U(C_t, H_t) = \log(C_t) - \tilde{B}H_t, \quad \tilde{B} > 0,$$

(4)

where

$$H_t = h_0 \int_{R_t}^{\infty} N_t(x) \, dx, \quad h_0 > 0,$$

(5)

and $\tilde{B}$ measures the marginal disutility of labor. The parameter $h_0$ denotes the constant fraction of total time endowment that each worker spends working when employed. The variable $R_t$ stands for the reservation productivity level above which job-matches exist and are employed. The variable $N_t(x)$ represents the fraction of the workforce that is characterized by the idiosyncratic productivity $x$. Hence, $H_t$ denotes total hours spent working.

Aggregate per capita output ($Y_t$) can either be used for private consumption, investment into new capital ($I_t$), or for advertising employment opportunities. It is assumed that posting a vacancy ($V_t$) comes at a constant advertising cost $a$, which is measured in terms of the single output good in the economy. The economy's aggregate resource constraint can then be expressed as the following inequality:

$$C_t + I_t + aV_t \leq Y_t.$$

(6)

Aggregate per capita output at any point in time is determined by a production function $F$ that uses as inputs the aggregate capital stock, $K_t$, and total hours worked measured in efficiency units, $H_t^e$:

$$Y_t = F(z_t H_t^e, K_t),$$

(7)

where

$$H_t^e = h_0 \int_{R_t}^{\gamma} \exp(x)N_t(x) \, dx.$$

(8)

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As I elaborate in Section 2.2.2, this utility function of the representative agent is derived by assuming that labor is indivisible, and that all agents in the economy can perfectly insure themselves against unemployment.
The function is assumed to be continuous and strictly monotonic in \( K_t \) and \( H^e_t \), and concave in \( K_t \) and \( H^e_t \) separately. In addition, it is assumed that the production function is homogeneous of degree one, and \( F(0,0) = 0 \). The technology shock \( z_t \) is assumed to be labor-augmenting. The per capita capital stock depreciates at the constant rate \( \delta_K \) in each period and increases by any investment undertaken. Thus, it obeys the following law of motion:

\[
K_{t+1} = (1 - \delta_K)K_t + I_t, \quad 0 \leq \delta_K \leq 1.
\]

The work force for every idiosyncratic level \( x \) above the reservation productivity level, \( R_t \), but below the upper end of the distribution, \( \gamma \), evolves according to

\[
N_t(x) = \begin{cases} 
(1 - \delta_N - \xi)N_t(x) + \xi g_xE_t & \text{if } x \in [R_t, \gamma), \\
0 & \text{otherwise}
\end{cases}
\]

where \( 0 \leq \delta_N \leq 1 \). The fraction of the work force concentrated at productivity level \( x \) in every period is determined by the flow of the work force to and from this level during that period. The exogenous portion of the flow away from productivity level \( x \) consists of two parts. The first part is the fraction of the work force at that level that retires at a constant rate \( \delta_N \), the second part is the fraction whose idiosyncratic productivity level changes with probability \( \xi \). The flow of the arriving work force consists of the fraction of all employed job-matches, \( E_t \), that changes its idiosyncratic productivity with probability \( \xi \) and is recategorized as level \( x \) with probability \( g_x \):

\[
E_t = \int_{R_t}^{\gamma} N_t(x) \, dx.
\]

Since \( \gamma \) constitutes a mass point at the upper end of the interval of idiosyncratic productivity levels, the law of motion of the fraction of the total work force concentrated at \( \gamma \) differs from its counterparts at all other levels:

\[
N_{t+1}(\gamma) = (1 - \delta_N - \xi)N_t(\gamma) + M_t, 0 < M_t,
\]

where \( M_t \) represents the number of new job-matches that are formed in time period \( t \). They can be thought of as being generated by the following Cobb–Douglas function that uses vacancies posted and total unemployment, \( U_t = 1 - E_t \), as inputs:

\[
M_t = \theta_t^{1-\lambda}U_t, \quad 0 \leq \lambda \leq 1, \quad \text{where } \theta_t = \frac{V_t}{U_t}.
\]

Hence, the fraction of the work force leaving productivity level \( \gamma \) is the same as the one at all other levels, but the stock of the employed work force at \( \gamma \) is
augmented by new job-matches only.\textsuperscript{3} No fraction of the employed work force whose idiosyncratic productivity level is recategorized during a period changes towards $\gamma$.

The matching technology $M_t$ implies an endogenous probability for the transition from unemployment to employment, $\tilde{m}_t$, and one for the transition from an unfilled to a filled job-vacancy, $m'_t$ each of which depends on the tightness of the labor market, $\theta_i$:

$$
\tilde{m}_t = \frac{M_t}{U_t} = \theta_t^{1-i}, \quad m'_t = \frac{M_t}{V_t} = \theta_t^{-i}.
$$

(14)

Furthermore, it implies that these transition probabilities decrease with an increasing degree of congestion, and vice versa.

With $N_t$ denoting the distribution of the available work force across idiosyncratic productivity levels in period $t$, the social planning problem consists of the planner choosing contingency plans for $\{C_t, V_t, R_t, K_{t+1}, N_{t+1} : t \geq 1\}$ at time 1 in order to maximize the objective function (3) subject to (1), (2), (6)-(13), $K_0, N_0$, and $z_0$. The social planner is assumed to make period $t$ decisions based on all the information available at time $t$. The timing is such that at the beginning of each period the planner inherits as state variables a capital stock, a distribution of the available work force across different labor productivity levels, as well as the previously realized technology shock. When an aggregate technology shock occurs, the planner decides upon the reservation productivity level that separates the available work force into the fraction that is employed and the fraction that is unemployed. Employment together with the existing capital stock produce aggregate output during the period. She also decides upon the level of investment in new capital, as well as in posting employment opportunities. Those posted vacancies determine the rate at which new job-matches are formed. Together with the recategorization of the work force that takes place as a consequence of shocks to idiosyncratic productivity which occur during the period, they determine the distribution of the work force across productivity levels at the beginning of the next period. Similarly, new investment adds to the existing capital stock, and, together with the fraction of the work force that will be employed, they become productive in the following period.

Assuming nonsatiation, the aggregate resource constraint is binding, the social welfare problem can be formulated as the following dynamic programming problem:

$$
W(\Omega_t) = \max_{\{C_t, V_t, R_t\}_{t=1}^{\infty}} \left[ \log(C_t) - \tilde{B}h_0 \int_{R_0} N_t(x) \, dx + \beta E_t W(\Omega_{t+1} | \Omega_t) \right]
$$

\textsuperscript{3} Taken together, (9) and (11) imply the law of motion for total employment, $E_t$, in the economy:

$$
E_{t+1} = (1 - \delta_N)E_t + M_t.
$$
s.t. $C_t = F(z_t, H_t, K_t) - I_t - aV_t,$

$$K_{t+1} = (1 - \delta_K)K_t + I_t,$$

$$N_{t+1}(x) = \begin{cases} (1 - \delta_N - \xi)N_t(x) + \xi g_xE_t & \text{if } x \in [R_t, \gamma), \\ 0 & \text{otherwise,} \end{cases}$$

$$N_{t+1}(\gamma) = (1 - \delta_N - \xi)N_t(\gamma) + M_t \quad \text{if } x = \gamma,$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}.$$ 

$\Omega_t$ denotes the aggregate state in time period $t$ that consists of the exogenous state variable $z_t$, and the endogenous state variables $N_t$ and $K_t$:

$$\Omega_t = \{z_t, N_t, K_t\}.$$ 

The corresponding Euler and costate equations shed some light on the planner’s allocation decision. They are used to show the equivalence between the social planner’s version and the decentralized market version of model economy:

$$I_t : 0 = -U_C + \beta E_z [W_{K_t} | \Omega_t] \frac{\partial K_{t+1}}{\partial I_t},$$

$$V_t : 0 = -aU_C + \beta E_z [W_{N_t}, (\gamma) | \Omega_t] M_V,$$

$$R_t : 0 = \{\tilde{B} - U_C, F_{N_t}(x) | x = R_t - \beta E_z [W_{N_{t+1}}, x = R_t | \Omega_t](1 - \delta_N - \xi)$$

$$- \beta \xi \int_{R_t}^{\gamma} E_z [W_{N_t(x')} | \Omega_t] g_{x'} \, dx'$$

$$- \beta E_z [W_{N_{t+1}}, (\gamma) | \Omega_t] M_{N_t(x) | x = R_t} \} N_t(R_t),$$

$$K_t : W_{K_t} = U_C, F_{K_t} + \beta E_z [W_{K_{t+1}} | \Omega_t] \frac{\partial K_{t+1}}{\partial K_t},$$

$$N_t(x) : W_{N_t(x)} = 1 \cdot \{R_t \leq x\} \{U_C, F_{N_t(x)} - \tilde{B} + \beta E_z [W_{N_{t+1}}(x) | \Omega_t](1 - \delta_N - \xi)$$

$$+ \beta \xi \int_{R_t}^{\gamma} E_z [W_{N_{t+1}}, x' | \Omega_t] g_{x'} \, dx' + \beta E_z [W_{N_{t+1}}, (\gamma) | \Omega_t] M_{N_t(x)} \} \forall x.$$ 

Note that the constant $h_0$ cancels from these equations. The expression $U_C$ denotes the derivative of function $U$ with respect to variable $C$. The remaining notation is analogous. These equations can be rearranged as follows:

(P1) $U_C = \beta E_z \{U_{C_{t+1}} [F_{K_{t+1}} + (1 - \delta_K)] | \Omega_t\},$

(P2) $a = \frac{\beta E_z [W_{N_{t+1}}, (\gamma) | \Omega_t]}{U_C} (1 - \lambda)m'_t,$
Eq. (P1) gives the standard condition for the optimal intertemporal allocation of consumption. Eqs. (P2) and (P3) denote the optimality conditions for creating new job-matches and destroying existing ones, respectively. They correspond to Eqs. (13) and (10), respectively, in Mortensen and Pissarides (1994). According to (P2), job-matches are created until the advertising cost of posting an additional vacancy just offsets the discounted expected future payoff from such a vacancy, measured in terms of the marginal utility of consumption. Assuming that the planner focuses on interior solutions with $N_i(R_i) > 0$, (P3) determines the optimal reservation productivity level $R_t$. The planner sets it so that the marginal product of the job-match employed at $R_t$ covers the opportunity cost of employment, $B$, plus the discounted expected social value resulting from new matches due to an increase in total unemployment. This expected social value is forgone if the reservation productivity level is not raised higher. The sum is corrected for the operational loss the planner is willing to incur now in anticipation of a future improvement in the quality of a match. This measure equals the option value of retaining an existing job-match. This option value consists of two different parts. First, it contains the discounted expected social value of the job-match characterized by $x = R_t$ that is neither retired at rate $\delta_N$ nor does it change its productivity level. If this expression is positive – possibly because $E_z[R_t+1|\Omega_t] < R_t$ – it pays to maintain the match today. Second, it contains the discounted expected social value of the match if its idiosyncratic productivity level improves. Eq. (P4) is an asset pricing equation. It corresponds to the asset value of a filled job with a given idiosyncratic productivity level that Mortensen and Pissarides (1994) state in their Eq. (5). For each productivity level $x$, it constrains the social value of employing a match at that level to equal the marginal product of the match, corrected for the opportunity cost of employment, the social value forgone due to job-matches that could not be created, and for the option value of the match. Note that at $x = R_t$, the asset value of employing a match equals zero.

2.2. The market economy

In the market version of my model economy, preferences, technology, the information structure, as well as the stochastic environment are assumed to be identical to their counterparts in the social planner’s version. Furthermore, firms and households are assumed to have rational expectations. That is, they make
their forecasts in a manner that is efficient and consistent with the equilibrium stochastic properties of the variables.

2.2.1. Firms' decision problem

Firms maintain a workforce \( n_t \) that is uniformly distributed across different idiosyncratic productivity levels \( x \) on the interval \( [-\gamma, \gamma] \). In each period, firms choose contingency plans for the amount of capital \( k_t \) that they rent from households and for the number of vacancies \( v_t \) that they post at a constant advertising cost \( a \) in order to make new hires. Furthermore, they choose contingency plans for the reservation level \( R_t \) that determines which of their job-matches will be employed and which will be unemployed. They do so in order to maximize the present discounted value of their future profit stream, subject to their production technology \( f \), factor prices, as well as \( m'_t \), the rate at which every vacancy posted leads to a job-match.

When discounting future profits, firms take into account the fact that households own the loanable funds needed to make investments in capital and in job-vacancies that they lend them at the interest rate \( \bar{R} \). The optimal amount of investment is determined by making households indifferent between consumption in two consecutive periods, i.e.,

\[
\hat{\beta}_t = \beta \frac{E_z[U_{C_{t+1}} | \Omega_t]}{U_{C_t}} = \frac{1}{1 + \bar{R}_t}.
\]

Each firm's production technology \( f \) has the same characteristics as the aggregate production function \( F \). Firms sell their output at a price normalized to one. They buy capital at the interest rate \( r \), and pay the wage rate \( w_x \) per hour worked to workers employed in a match with productivity level \( x \). Thus, the problem faced by each firm can be summarized by the following dynamic programming problem:

\[
W^F(\omega^F_t) = \max_{(\omega^*_t, k_t, \gamma^*)} \left\{ f(z_t, h^*_t, k_t) - \int_{R_t}^\gamma w_x h_0 n_t(x) \, dx - r k_t - a v_t + \hat{\beta}_t E_z[W^F(\omega^F_{t+1} | \omega^F_t)] \right\}
\]

s.t. \( n_{t+1}(x) = \begin{cases} (1 - \delta_N - \xi) n_t(x) + \xi g_x \int_{R_t}^\gamma n_t(x') \, dx' & \text{if } x \in [R_t, \gamma), \\ 0 & \text{otherwise}, \end{cases} \)

\( n_{t+1}(\gamma) = (1 - \delta_N - \xi) n_t(\gamma) + m'_t v_t \) if \( x = \gamma \),

\( N_{t+1}(x) = \begin{cases} (1 - \delta_N - \xi) N_t(x) + \xi g_x \int_{R_t}^\gamma N_t(x') \, dx' & \text{if } x \in [R_t, \gamma), \\ 0 & \text{otherwise}, \end{cases} \)
\[ N_{t+1}(\gamma) = (1 - \delta_N - \xi)N_t(\gamma) + M_t \quad \text{if} \quad x = \gamma, \]

\[ z_{t+1} = \rho z_t + \varepsilon_{t+1} \quad \text{with} \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2_\varepsilon), \]

where \( w_t = w_t(x, \Omega_t), \) \( r_t = r_t(\Omega_t), \) \( \omega_F^t = \{z_t, n_t, N_t\}. \)

The following Euler and costate equations correspond to this problem:

\( (F1) : f_{k_t} = r_t(\Omega_t), \)

\( (F2) : a = \frac{\beta U_{c_{t+1}} E_x[W_{n_{t+1}(\gamma)}]|_{\omega_F^t}}{U_{c_{t}}} m_t', \)

\( (F3) : f_{n_t(x)}|_{x=R_t} = w_{R_t} - \beta_t E_z[W_{n_{t+1}(x)}|_{x=R_t}|_{\omega_F^t}](1 - \delta_N - \xi) \)

\[ -\beta_t^t \xi \int_{R_t}^{\gamma} E_z[W_{n_{t+1}(x')}|_{\omega_F^t} dx'], \]

\( (F4) : W_{n_t(x)}^F = 1 \cdot \{R_t \leq x\} \{f_{n_t(x)} - w_x + \beta_t E_z[W_{n_{t+1}(x)}|_{\omega_F^t}](1 - \delta_N - \xi) \)

\[ +\beta_t^t \xi \int_{R_t}^{\gamma} E_z[W_{n_{t+1}(x')}|_{\omega_F^t} dx'] \quad \forall x. \]

Note that the constant \( h_0 \) cancels from Eqs. (F2)-(F4).

These Euler and costate equations can be interpreted as follows. (F1) describes the classical result that firms rent capital from households up to the point where the marginal product of the last unit added equals the real rate of interest. Eqs. (F2) and (F3) give the optimality conditions for posting vacancies and for determining the reservation productivity level, respectively. (F2) corresponds to the zero-profit condition for new entrants, and (F3) to the job-destruction condition in the MP-framework. According to (F2), the marginal vacancy is characterized by the fact that the flow cost of advertising equals the expected flow benefit. Eq. (F3) states that the marginal job-match employed is characterized by its marginal product covering the per-hour wage rate, corrected for the option value of the match. Finally, (F4) corresponds to an asset pricing equation that determines the dynamically optimal allocation of job-matches across productivity levels. Note that at \( x = R_t, \) the value to the firm of employing a job-match equals zero. Mortensen and Pissarides (1994) use this fact to determine the optimal reservation productivity level in their Eq. (10).

### 2.2.2. Households' decision problem

By assumption, there is a complete market of state contingent claims traded among households at time zero which pays off one unit of the consumption good in every possible state of the world at time \( t. \) Let \( S_t \) denote the space of all conceivable individual labor market histories, \( s_t, \) up to time \( t, \) where \( s_t \) denotes the set of pairs of the random variables \( x \) and \( z \) that have been realized.
for an individual up to time $t$: $s_t = [(x_1, z_1), (x_2, z_2), \ldots, (x_t, z_t)] \in S_t$. Note that $s_t = [s_{t-1}, (x_t, z_t)] \forall t$. The space $S_t$ can be denoted as follows:

$$S_t = ([-\gamma, \gamma] \times \mathbb{R})^t.$$

It equals the cross product of all conceivable individual labor market histories.

Due to the assumption of rational expectations, the measure of each possible history $\mu(s_t)$ is known at time zero. As time unfolds, individuals learn their true histories. Different histories are associated with different wealth. Therefore, individuals would like to insure themselves at time zero against the various histories. A complete market involves a full set of claims $\pi_t(s_t)$ contingent on the potential history:

$$\pi_t(s|s_t) = \begin{cases} 1 & \text{if } s = s_t, \\ 0 & \text{if } s \neq s_t, \end{cases}$$  \hspace{1cm} (15)

Hence, a claim $\pi_t(s|s_t)$ pays off one unit of the consumption good at time $t$ if and only if the state $s_t$ is realized in period $t$. The period zero price of such a claim is denoted by $q_t(x_t, z_t)$, and the period zero price of a consumption good that is consumed in period $t$ is denoted by $p_t(x_t, z_t)$. Abbreviate $\pi_t(s|s_t)$ as $\rho_t(s_t)$, and let $e_t(x_t, z_t)$ and $u_t(x_t, z_t)$ denote each household’s probability of being employed in period $t$, or unemployed, respectively. Note that each individual’s average probability of being employed in period $t$ or unemployed, respectively, equals the macroeconomic pendants:

$$\int_{-\infty}^{\infty} \int_{-\gamma}^{\gamma} e_t(x_t, z_t) g_x \, dx \, dz = \int_{R_t} N_t(x) \, dx,$$

$$\int_{-\infty}^{\infty} \int_{-\gamma}^{\gamma} u_t(x_t, z_t) g_x \, dx \, dz = 1 - \int_{R_t} N_t(x) \, dx.$$

(16)

As owners of the capital stock households choose contingency plans for consumption, for insurance claims, and for capital as a function of possible realizations of their state in the labor force. They do so in order to maximize the present discounted value of their expected lifetime utility subject to their lifetime budget constraint, the laws of motion for capital, as well as for the aggregate technology shock, where $h_z$ denotes the density function of the normal distribution. Each household’s total time endowment is assumed to be fixed and normalized to one. A household can either work for $h_t$ hours or enjoy leisure $l_t$:

$$1 = h_t + l_t.$$  \hspace{1cm} (17)

When employed, each worker is assumed to work a constant shift length $h_0$ which is measured in terms of total time endowment. If a worker is unemployed, he gets to enjoy leisure, so that $l_t = 1$. I let $(\cdot)$ denote $(x, z)$ and $d_t$ any profits that are distributed from firms to households in the form of dividends. Furthermore, $v_t$
represents the Lagrange multiplier of the resource constraint in period $t$. Following Hansen (1985), I assume that each household’s preferences are represented by a utility function that is additively separable in consumption and hours spent working:

$$U(c_t, h_t) = \log(c_t) + \tilde{A} \log(1 - h_t) \quad \tilde{A} > 0. \quad (18)$$

Hence, in every period $t$, each worker’s expected utility is given by

$$E[U(c_t, h_t)] = \int_{R_t} N_t(x) dx [\mu_{t-1}(\cdot) \int_{-\infty}^\gamma \log(c_t(\cdot)) + \tilde{A} \log(1 - h_0) g_x \, dx h_x \, dz]$$

$$+ [1 - \int_{R_t} N_t(x) dx] [\mu_{t-1}(\cdot) \int_{-\infty}^\gamma \log(c_t(\cdot)) + \tilde{A} \log(1 - h_0) g_x \, dx h_x \, dz].$$

This expression can be rewritten as

$$E[U(c_t, h_t)] = \mu_{t-1}(\cdot) \int_{-\infty}^\gamma \log(c_t(\cdot)) + e_t(\cdot) \tilde{A} \log(1 - h_0) g_x \, dx h_x \, dz. \quad (19)$$

I define $\tilde{B} = -\tilde{A} \log(1 - h_0)/h_0$. Note that in each state of the world, the fraction of total hours worked, $h_t(\cdot)$, can be expressed as the product of the fraction of households that are employed and the constant shift length $h_0$:

$$h_t(\cdot) = h_0 e_t(\cdot). \quad (20)$$

Making use of Eq. (17), Eq. (19) can be reformulated as

$$E[U(c_t, h_t)] = \mu_{t-1}(\cdot) \int_{-\infty}^\gamma \log(c_t(\cdot)) - \tilde{B} h_t(\cdot) g_x \, dx h_x \, dz. \quad (21)$$

Then, each household’s optimization problem can be summarized as follows:

Problem A:

$$\max \sum_{t=1}^{\infty} \beta_t \mu_{t-1}(\cdot) \int_{-\infty}^\gamma \log(c_t(\cdot)) - \tilde{B} h_t(\cdot) g_x \, dx h_x \, dz$$

s.t. (1):

$$\sum_{t=1}^{\infty} \int_{-\infty}^\gamma \int_{-\infty}^\gamma p_t(\cdot) [c_t(\cdot) + i_t(\cdot) + q_t(\cdot) \pi_t(\cdot)] g_x \, dx h_x \, dz$$

$$\leq \sum_{t=1}^{\infty} \int_{-\infty}^\gamma \int_{-\infty}^\gamma p_t(\cdot) [w_t(x_t, \Omega_t) h_0 e_t(\cdot)$$

$$+ r_t(x_t, \Omega_t) k_t(\cdot) + \pi_t(\cdot) + d_t] g_x \, dx h_x \, dz,$$
Proposition 2.1. Any solution to problem A implies the solution to an alternative formulation of the household's problem that can be restated as problem B.

Problem B:

\[
W(\omega^H) = \max \left\{ \log(c_t) - \tilde{h}_t \int_{R_t} n_t(x) \, dx + \beta E_z[W^H(\omega^H_{t+1} | \omega^H_t)] \right\}
\]

s.t. (1) \[ \sum_{t=1}^{\infty} p_t(c_t + i_t) \leq \sum_{t=1}^{\infty} p_t[w_t(\Omega_t)h_0 \int_{R_t} n_t(x) \, dx + r_t(\Omega_t)k_t + d_t]. \]

(2) \[ k_{t+1} = (1 - \delta_k)k_t + i_t, \]

(3) \[ K_{t+1} = (1 - \delta_k)K_t + I_t, \]

(4) \[ z_{t+1} = \rho z_t + \epsilon_{t+1} \] with \( \epsilon_t \sim N(0, \sigma^2_\epsilon), \)

where \( w_t(\Omega_t) = \int_{-\gamma}^{\gamma} w_t(x_t, \Omega_t) \, dx, \) \( r_t(\Omega_t) = \int_{-\gamma}^{\gamma} r_t(x_t, \Omega_t) \, dx, \) and \( \omega^H = \{z_t, k_t, K_t\}. \)

Euler equations of problem A:

(1) \[ \mu(x_{t-1}, z_{t-1})U_{c_t}(\cdot) - p_t(\cdot) = 0, \]

(2) \[ \nu_t p_t(\cdot) - \nu_{t+1} p_{t+1}(\cdot) - (1 - \delta_K) = 0, \]

(3) \[ \nu_t p_t(\cdot)[1 - q_t(\cdot)] = 0; \Rightarrow q_t(\cdot) = 1 \text{ for an interior solution} \]

(4) \[ \sum_{t=1}^{\infty} \int_{-\gamma}^{\gamma} p_t(\cdot)\{c_t(\cdot) + i_t(\cdot) + q_t(\cdot)\pi_t(\cdot)\}g_t \, dx \, dz \]

\[ = \sum_{t=1}^{\infty} \int_{-\gamma}^{\gamma} p_t(\cdot)\{w_t(x_t, \Omega_t)h_0 e_t(\cdot) \]

\[ + r_t(x_t, \Omega_t)k_t(\cdot) + \pi_t(\cdot) + d_t\}g_{t+1} \, dx \, dz \]

\[ = \frac{\mu(x_{t-1}, z_{t-1})U_{c_t}(\cdot)}{\mu(x_{t-1}', z_{t-1}')U_{c_t}(\cdot')} \]

\[ \frac{p_t(\cdot)}{p_t(\cdot')} = \frac{\mu(x_{t-1}, z_{t-1})g_{t+1}h_{t+1}}{\mu(x_{t-1}', z_{t-1}')g_{t+1}h_{t+1}}. \]

The very last equality results from the fact that, in the optimum, households cannot engage in profitable arbitrage across different states. Hence,

\[ U_{c_t(x, z)} = U_{c_t(x', z')} \forall (x, z), \ (x', z') \in [-\gamma, \gamma] \times \mathbb{R}, \] (22)
and consumption in any time period is independent of the individual labor market history. Eq. (22) determines optimal risk-sharing among risk-averse agents as in Arrow (1971) and Borch (1962). It follows that with preferences separable in consumption and leisure, households can completely insure themselves against any idiosyncratic income risk.

Euler equations of problem B:

\[ i_t : U_c(t) - \beta E_t [U_c(H_t + 1, I_t + 1) + (1 - \delta_t)] \omega_t^H \frac{\partial k_{t+1}}{\partial i_t}, \]

\[ k_t : W^H_{k_t} = U_c(r_t(\Omega_t) + \beta E_t [W^H_{k_t+1} \omega_t^H] \frac{\partial k_{t+1}}{\partial k_t}, \]

\[ v_t : \sum_{t=1}^{\infty} p_t[c_t + i_t] \leq \sum_{t=1}^{\infty} p_t[w_t(\Omega_t)h_0 \int_0^y N_t(x) dx + r_t(\Omega_t)k_t + d_t]. \]

They can be summarized as follows:

(H1): \( U_c - \beta E_t [U_c(r_{t+1}(\Omega_{t+1}) + (1 - \delta_t))] \omega_t^H = 0, \)

(H2): \( \sum_{t=1}^{\infty} p_t[c_t + i_t] \leq \sum_{t=1}^{\infty} p_t[w_t(\Omega_t)h_0 \int_0^y N_t(x) dx + r_t(\Omega_t)k_t + d_t]. \)

Given complete insurance as well as the result that \( q_t(x_t, z_t) = 1 \) in any optimum, it follows from the household’s lifetime budget constraint in problem A that \( i_t(x_t, z_t) = i_t(x_t^I, z_t^I) \) \( \forall x_t, x_t^I \in [-\gamma, \gamma] \). The lifetime budget constraints are identical in problem A and problem B. So are the optimal paths of consumption and investment. Furthermore, according to Walras’ law, the market for financial claims clears whenever the markets for goods and factors of production clear. Hence, any solution to problem A does imply the solution to problem B.

2.3. A recursive competitive equilibrium

Given the structure of this decentralized economy, a recursive competitive equilibrium is defined as follows. All agents in the economy solve their constrained maximization problem by taking as given the equilibrium factor prices, the equilibrium rate at which firms’ search effort leads to a job-match, as well as the laws of motion for the individual and aggregate state variables. Furthermore, all markets clear, and the laws of motion of the representative consumer’s endogenous state variables result when aggregating the individuals’ laws of motion. Finally, the individual first-order conditions that are necessary for an optimum coincide with the first-order conditions for the representative agent. This recursive competitive equilibrium decentralizes the social planning problem in the sense of yielding identical allocations.

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\(^4\)The notion of a recursive competitive equilibrium is developed in Prescott and Mehra [1980].
The following factor prices \( w_n, r, \) and matching rate \( m'_t \) make the individual Euler conditions coincide with the planner's Euler condition for the representative household:

\[
w_n = \lambda[F_{N_t(x)} + a \frac{V_t}{U_t}] + (I - \lambda) \frac{\dot{\bar{B}}}{U_C}, \quad \forall x \in [R_t, \gamma],
\]

\[
r_t = F_k,
\]

\[
m'_t = \frac{M_{vt}}{1 - \lambda}.
\]

This can be checked as follows. Substituting the equilibrium interest rate that results from (F1) into (H1) yields (P1). The equilibrium matching rate \( m'_t \), together with (F2), yield (P2). Similarly, when substituting the equilibrium real wage rate into (F4) and combining the outcome with (P2), (P4) results. Finally, when substituting the equilibrium wage rate into (F3) and combining the outcome with (P2) and (F4), (P3) results. Since the model includes identical households and firms, and since all markets clear, \( K_t = k_t, I_t = i_t, C_t = c_t, V_t = v_t \) and \( N_t = n_t \) for all \( t \) in equilibrium. Furthermore, since the Euler equations for the social planning problem coincide with the ones of the firms' and households' dynamic optimization problem, so do their equilibrium allocations.

In this economy, capital is assumed to be a homogeneous good; it can instantaneously be moved across different job-matches. It is not surprising that the standard neoclassical outcome - that the equilibrium interest rate equals the marginal product of capital - holds in this framework as well. Since there is a continuum of job-matches that differ in their idiosyncratic productivity level, there is also a continuum of equilibrium wage rates. These wage rates equal the weighted average between the sum of the marginal product of labor and the total advertising costs per number of unemployed workers, and the opportunity cost of employment measured in terms of the marginal utility of consumption. These two points can be thought of as the threat points of a wage bargaining process which takes place between the worker and the firm once they have created a new job-match. The worker asks for his marginal contribution to the production of output plus advertising costs forgone due to the creation of the match, while the firm offers to pay the worker his reservation wage. The actual mix of these two threat points depends on the size of \( \lambda \) - the elasticity of the matching function with respect to unemployment - which can be interpreted as the households' bargaining power. The vector of wage rates stated above is an equilibrium vector for any possible value of \( \lambda \). Also, the bargaining process is repeated in every period for as long as the job-match exists.

Under recursive competitive equilibrium, individual households and firms take the equilibrium wage rates as given when solving their optimization problems. Their actions generate the vector of equilibrium wage rates. However, the vector of wage rates necessary to guarantee a Pareto optimal allocation will not equal
negotiated rates, unless incentives are set correctly. There is nothing inherent in the model which guarantees that these two vectors coincide. Moen (1993) addresses the issue of implementing efficient wage rates. He suggests an alternative structure for the labor market by assuming that firms can communicate wage offers to potential workers before they are matched by announcing the accompanying offered wage when posting a vacancy. In this case, the equilibrium wage offer leads to an efficient allocation of resources.

Apart from factor prices, firms also take as given the rate at which their advertising activity leads to a job-match when solving their constrained maximization problem. In equilibrium, this rate equals the one that results from all agents' actions. Given that agents are assumed to have rational expectations, this rate is determined so that each firm's weighted marginal contribution to creating a job-match corresponds to its average contribution, the weight being equal to the elasticity of the aggregate matching technology with respect to vacancies. Taken together, the equilibrium factor prices and the matching rate for firms generate an equilibrium that is Pareto optimal. Pareto optimality is achieved in the labor market, since search externalities just offset each other.

3. Conclusions

In this paper I have provided one possible microeconomic structure that is needed to reformulate the social planner's problem of a generalized version of the MP-framework as a decentralized market economy. I have shown that for an indivisible labor model with persistent unemployment, where households have access to a complete insurance market before any other economic activity starts, they can fully insure themselves against variations in wealth. This generalizes the result derived by Rogerson (1988) and Hansen (1985) with a simple lottery for a framework with nonpersistent unemployment to one in which unemployment persists. I have also derived the vector of match-specific wage rates, an interest rate and a matching rate for firms that support the Pareto optimal outcome of the social planner's problem as a recursive competitive equilibrium.

This planning problem, its assumptions and results can be regarded as a benchmark comparison for results derived for models using less stringent assumptions.

Appendix. A sketch of the labor market in the model economy

The circle in Fig. A.1 depicts the constant labor force that is normalized to one. It contains all employed job-matches and the pool of unemployment. Job-matches are employed as long as their match-specific productivity lies above the reservation productivity level. Otherwise they are dissolved, and the worker
involved is sent to the pool of unemployment. All job-matches retire with a constant probability $\delta_N$.

The diagram on the right-hand side depicts the distribution of job-matches across all idiosyncratic productivity levels ranging from the reservation productivity to the upper end of the support, $\gamma$. The distribution is generated by idiosyncratic productivity shocks that hit all job-matches in every period and that may change their idiosyncratic labor productivity. A match is dissolved whenever its idiosyncratic productivity level drops below the reservation productivity. It happens if the match draws a bad idiosyncratic shock that lowers its labor productivity, and/or if the reservation productivity rises beyond the match’s given idiosyncratic productivity level.

New job-matches are generated by the number of job-vacancies posted and the size of the pool of unemployment. By assumption, new matches occur at the highest productivity level, $\gamma$.

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