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Heterogeneous job-matches and the cyclical behavior of labor turnover

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Abstract

Worker flows to and from unemployment simultaneously occur over the US business cycle, and the size of the flows is positively linked to the unemployment rate. Unemployment flows and the unemployment rate are highly volatile, persistent and countercyclical. Inflows lead the unemployment rate, and the unemployment rate leads outflows over the business cycle. The one-sector stochastic growth model is augmented by matching frictions in the labor market and match-specific productivity shocks that introduce *ex post* heterogeneous job-matches. Matching frictions help generate the lead–lag relationship between unemployment flows and the unemployment rate. Combined with heterogeneous job-matches they generate endogenous unemployment flows and an unemployment rate whose dynamic characteristics match observed data. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The dynamic behavior of the unemployment rate is determined by the worker flows into and out of unemployment. Individuals become unemployed because they are laid off, they quit their job, or they enter the labor force before finding a job. Individuals leave the unemployment pool because they are newly hired,

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recalled, or because they become discouraged and leave the labor force. To understand the movement of the unemployment rate over time, one needs to understand the movement of the unemployment flows and their underlying driving forces. Davis et al. (1996) provide ample empirical evidence for the close ties between the dynamic behavior of the unemployment flows and that of the unemployment rate. Their worker flows are computed from the Bureau of Labor Statistics' data on unemployment duration by reason for unemployment.[1] Fig. 1(a) depicts US net flows to unemployment which are labeled *inflows* and their various subflows. Fig. 1(b) does the same for net flows from unemployment. The sum of flows is labeled *outflows*. Selected statistics on the flow series and the unemployment rate are reported in Table 1.

In this paper I explore the extent to which matching frictions in the labor market combined with heterogeneity in job-matches can explain the dynamics of unemployment flows and the unemployment rate. I develop a dynamic general equilibrium model which explicitly features new hires, recalls, and temporary layoffs. I assume permanent layoffs and quits into unemployment are a constant fraction of employment. Capturing these subflows allows me to study the role that each flow plays in generating the volatility, persistence, and cyclicality of total unemployment flows and the unemployment rate. I augment the one-sector stochastic growth model as presented by Kydland and Prescott (1982) by two features: matching frictions in the labor market and match-specific productivity shocks. I define a job-match as a combination of a worker and a job. Matching frictions imply that it takes time to hire a new worker. These frictions help generate the lead–lag relationship between various unemployment flows and the unemployment rate. Match-specific shocks introduce *ex post* heterogeneity across job-matches. Together with matching frictions these shocks allow for endogenous layoffs and hires. Heterogeneity and the implied endogenous layoffs are one possibility to generate flows into unemployment that are highly volatile, thereby replicating the empirical observation that inflows – rather than outflows – are the key driving force behind the cyclical variation in the unemployment rate (see Davis et al. (1996), (p. 137)). Endogenous layoffs can be expected to significantly improve the performance of a model with homogeneous job-matches and layoffs that are a constant fraction of employment. Such a model is developed by Merz (1995). Her model counterfactually predicts that outflows are the driving force underlying movements in the unemployment rate, and that inflows are procyclical. The model presented here can be thought of as combining elements of the Mortensen–Pissarides model, as

[1] Appendix A describes data sources and how the worker flow data were constructed. Davis et al. (1996), (p. 131) discuss advantages and disadvantages of constructing unemployment flows from unemployment duration data or from gross flow data. They report the correlation between inflows constructed from the two data sets to equal 0.92. The correlation between outflows is equally high.

Table 1
US unemployment flows by cause and unemployment rate 1977:2–1996:4

Panel A. Autocorrelations

Statistic	1	2	3
$\rho(in_t, in_{t-\tau})$	0.793	0.541	0.341
$\rho(tl_t, tl_{t-\tau})$	0.694	0.340	0.146
$\rho(pl_t, pl_{t-\tau})$	0.704	0.555	0.460
$\rho(q_t, q_{t-\tau})$	0.731	0.448	0.251
$\rho(ent_t, ent_{t-\tau})$	0.857	0.651	0.454
$\rho(out_t, out_{t-\tau})$	0.803	0.618	0.450
$\rho(rc_t, rc_{t-\tau})$	0.675	0.395	0.155
$\rho(m_t, m_{t-\tau})$	0.767	0.593	0.449
$\rho(u_t, u_{t-\tau})$	0.904	0.724	0.518

Panel B. Dynamic cross correlations

	τ				
Statistic	− 2	− 1	0	1	2
$\rho(in_t, u_{t-\tau})$	0.796	0.876	0.825	0.585	0.348
$\rho(tl_t, u_{t-\tau})$	0.678	0.675	0.541	0.221	− 0.076
$\rho(pl_t, u_{t-\tau})$	0.759	0.856	0.866	0.678	0.495
$\rho(q_t, u_{t-\tau})$	− 0.260	− 0.411	− 0.563	− 0.622	− 0.620
$\rho(ent_t, u_{t-\tau})$	0.507	0.643	0.717	0.708	0.646
$\rho(out_t, u_{t-\tau})$	0.603	0.758	0.853	0.848	0.737
$\rho(rc_t, u_{t-\tau})$	0.669	0.724	0.684	0.513	0.237
$\rho(m_t, u_{t-\tau})$	0.386	0.542	0.678	0.763	0.773

Notes: $\rho(x_t, y_{t-\tau})$ denotes the correlation between variable x and the τth lag (lead) of variable y if τ is positive (negative). *in* denotes inflows to unemployment. They consist of temporary layoffs, *tl*, permanent layoffs, *pl*, quits, *q*, and entrants to the labor force, ent. *out* denotes outflows from unemployment. Outflows consist of recalls, *rc*, and new job-matches, m. u denotes the unemployment rate. Flows are constructed as described in Appendix A.

developed by Mortensen and Pissarides (1994, 1993), and the standard stochastic growth model.

Fig. 1(a) and 1(b) in conjunction with Table 1 paint a clear picture of the empirical facts I aim to explain: The cyclical variability of unemployment inflows is primarily determined by layoffs which on average constitute 48% of all inflows. 40% of all layoffs are temporary. Entrants to the labor force and quits primarily affect the level of total inflows and their persistence. New hires

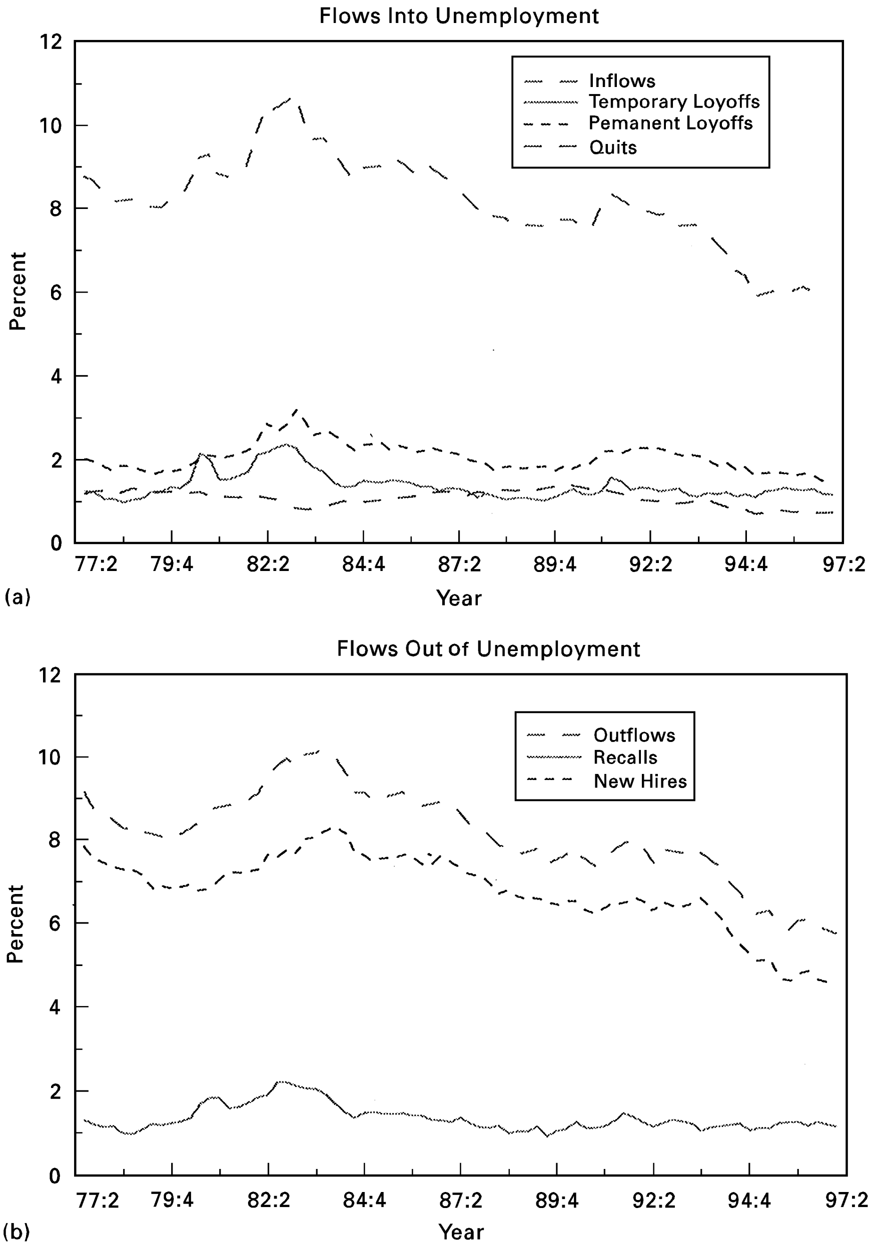

Fig. 1. (a) Flows into unemployment, (b) flows out of unemployment
Source: Author's own calculations based on unemployment duration data for reason of unemployment from the Current Population Survey. The difference between total inflows and the sum of permanent layoffs, temporary layoffs and quits is due to entrants to the labor force.

and recalls determine the movement of outflows. New hires constitute 83% of all outflows. For most unemployment flows there tends to be an inverse link between volatility and persistence. For example, temporary layoffs are more volatile and less persistent than inflows which, in turn, are more volatile and less persistent than permanent layoffs. The ranking is the same for recalls, outflows, and new hires. Also, inflows are more volatile and less persistent than outflows. Inflows and outflows are countercyclical and so are their respective subflows with the exception of procyclical quits. Except for quits the various inflows lead the unemployment rate over the business cycle. The unemployment rate leads outflows which results from recalls leading and new hires following unemployment over the business cycle.

In this paper I refrain from explicitly modeling permanent layoffs and quits. I do so mostly for technical reasons. With heterogeneous job-matches, the match-distribution enters the state space. Permanent layoffs and quits would destroy the second-order differentiability of Euler equations at every point in the state space by introducing a discontinuity at the reservation productivity level. This phenomenon would call for solution procedures that are considerably less tractable than the linearization method I apply.[2] For the same reason I assume that temporarily laid off workers cannot search for a new job. They need to wait to be recalled in order to reenter employment. This assumption seems justified because, by definition, a layoff is labeled temporary only if the worker is actually recalled. Furthermore, Katz and Meyer (1990) provide evidence that the recall rate of layoffs that originally are intended to be temporary amounts to 80 percent. I also assume a constant labor force in order to focus the analysis on flows between unemployment and employment. According to studies of gross labor turnover by Blanchard and Diamond (1990), (p.92) and Burda and Wyplosz (1994) these flows constitute a fraction of all unemployment flows large enough to deserve analytical attention. This assumption helps avoid potentially counterfactual simulation results. For example, if workers can move from out of the labor force into search unemployment when economic conditions improve, an increase in unemployment is likely to coincide with an increase in job-vacancies. This would contradict a well known empirical regularity – the negative correlation between vacancies and unemployment.

The key variables transmitting exogenous shocks and generating endogenous unemployment flows are the countercyclical reservation productivity level and procyclical job-vacancies. The reservation productivity determines temporary layoffs and recalls, and job-vacancies affect new hires. The simulation results

[2] Den Haan et al. (1997) have only recently mastered the technical challenge of explicitly modeling permanent layoffs in a framework similar to the one presented here. They rely on a Gaussian Hermite quadrature approach.

suggest the model performs well in generating the observed volatility, persistence, cyclicality and comovement of unemployment flows and the unemployment rate. The results correctly identify temporary layoffs and recalls as primary determinants of the flows' variability. Permanent layoffs and new hires are important for generating the observed persistence. The interaction between the various subflows generates the timing relationship between unemployment flows and the unemployment rate. I argue that explicitly modeling permanent layoffs will enhance the results presented.

The paper is organized as follows. In Section 2, I describe the economic environment of my model economy. I present the social planner's version and study the accompanying Euler conditions. In Section 3 I derive expressions for unemployment flows, and in Section 4, I use these flows to derive steady state measures of the incidence and duration of unemployment. Section 5 describes how the parameters used to calibrate the model are chosen. I present and discuss the simulation results in Section 6 and end with conclusions in Section 7.

2. The model

The analysis that follows is based on a one-sector stochastic growth model with matching frictions in the labor market and job-matches that are recurrently exposed to both idiosyncratic and aggregate technology shocks. This framework generates flows to and from unemployment that I aim to explore.

2.1. The economic environment

The stylized economy is populated by a continuum of infinitely-lived worker-consumers with names on the closed interval [0,1] and a continuum of identical firms. Workers and firms exchange goods and factors of production. Firms produce output via a constant-returns-to-scale technology using labor and physical capital as inputs. Output goods are either consumed or invested. The markets for goods and capital are perfectly competitive. Matching frictions characterize the labor market. These frictions capture the idea that it takes time and resources to create new job-matches. They are represented by a matching technology which turns vacancies and unmatched workers into new job-matches. In each period job-matches are exposed to aggregate and to idiosyncratic shocks to productivity. Idiosyncratic shocks introduce heterogeneity among job-matches and allow for endogenous temporary layoffs and recalls. Job-matches terminate with a constant probability.

The events of the labor market are sketched in Fig. 3 in Appendix B. This figure depicts various states in which worker-consumers can be found and the various decisions and exogenous forces that make them move across these states. All households are members in the constant labor force, L, that is normalized to one. The constant work force assumption implies that there is no labor–leisure choice in the model. Worker-households differ with respect to their status in the labor force. At any point in time, workers can be engaged in a job-match and employed, E, or they can participate in the unemployment pool, U. When employed, all workers work a constant shift-length, h_0, implying that all variations in labor input are due to variations in individuals' employment status. Each match is assumed to face a constant per-period probability, δ_N, of being dissolved with the worker becoming permanently unemployed. This assumption captures the flow of workers out of employment which occurs independently from a firm's temporary layoff decision such as permanent layoffs, or quits into unemployment.

All workers are identical *ex ante*. Once they are engaged in a job-match, their match is exposed to recurring aggregate productivity shocks, and also to idiosyncratic shocks which may change the match's idiosyncratic labor productivity. *Ex post* heterogeneity exists among job-matches, and firms endogenously decide which job-match to temporarily lay off and which to employ. Job-matches can be temporarily laid off for two separate reasons. Either the match is laid off because an idiosyncratic shock decreases its productivity below the firm's reservation productivity, R, or because the reservation productivity increases beyond the match's given labor productivity. Of course, a job-match can be laid off if both events occur simultaneously. If temporarily unemployed, workers can return to employment only by being recalled. By assumption, temporarily laid off workers cannot search. If permanently unemployed, a worker faces a certain chance of being rematched with another firm, changing the worker's status from unemployed to employed. This probability varies with the tightness of the labor market measured by the total number of vacancies, V, relative to the total number of permanently unemployed workers. It increases with an increase in the market thickness brought about by a relative increase in the number of listed job-vacancies. It decreases with congestion caused by a relative increase in permanent unemployment.

All of this amounts to saying that firms can vary their level of employment in several ways. They can hire workers from the pool of permanent unemployment. In order to attract applicants and to create a new job-match, M, they need to post vacancies. Posting vacancies comes at an advertising cost. The productivity of this new job-match is drawn from a uniform distribution. This specification captures Jovanovic's (1979) idea that the quality of a new job-match often is revealed to the firm only after it has been created. Alternatively, the firm can recall a worker from the pool of temporary layoffs. In terms of Fig. 3, recalls are represented by job-matches moving from the state of temporary layoffs to employment. This occurs either because firms lower their reservation productivity, or because a temporarily laid off match receives an idiosyncratic shock which takes it above the cutoff line. Such a recall comes at no cost. A firm can decrease its level of employment by temporarily laying off workers. It chooses to do so whenever a worker's idiosyncratic productivity level drops below the firm's reservation productivity.

I assume labor to be indivisible. Households can either be employed or unemployed which introduces nonconvex consumption possibility sets. Since different labor market histories are associated with different wealth levels, households have an incentive to insure themselves against those various histories. In Merz (1997) I show that if they have access to a complete insurance market, they fully insure themselves against any income risk arising from the possibility of becoming unemployed. Furthermore, I derive the equilibrium values of the vector of wage rates, the interest rate, as well as the matching rate for job-vacancies that are required to insure that all matching externalities just offset one another, thereby making the economic environment Pareto optimal. I then invoke the second welfare theorem to reformulate the market economy as a social planning problem. In this paper I present the social planner's version of my model economy. It specifies preferences, technologies, economic constraints, the stochastic environment, and the information structure.

2.2. The social planning problem

Heterogeneity in the labor market is introduced via the assumption that existing job-matches are exposed to an idiosyncratic productivity shock orthogonal to a common aggregate technology shock. The aggregate technology shock z_t is assumed to follow an AR(1) process with the following law of motion:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad 0 < \rho < 1, \tag{2.1}$$

where ε_t is an i.i.d. random variable drawn from a normal distribution with mean zero and standard deviation σ_ε. Shocks to the idiosyncratic productivity level x_t follow a stochastic process that is uniformly distributed across the interval $[-\gamma,\gamma]$:

$$x_{t+1} = \begin{cases} x_t & \text{with probability } (1-\xi) \\ x \sim G\colon[-\gamma,\gamma] \to [0,1] & \text{with probability } \xi g_x, \end{cases}$$

where $g_x = G'(x)$. (2.2)

This process implies the following probability density function g_x and cumulative distribution function $G(x)$ for the random variable x:

$$g_x = \frac{1}{2\gamma}, \qquad G(x) = \frac{x+\gamma}{2\gamma}. \tag{2.3}$$

Hence, x_t is independently and identically distributed across job-matches, first-order Markov, and positively autocorrelated with bounded support. With probability $(1-\xi)$, a job-match is characterized by the same idiosyncratic productivity in two consecutive periods. With probability ξ, this productivity level will change. In that case, the new level will be independent from what it is initially, i.e., draws from the distribution G are i.i.d. Furthermore, I assume newly created job-matches to be uniformly distributed across the range of all possible productivity levels. This structure captures the idea that the match-quality often times is revealed only after the match has been formed. Depending on the quality, the match is either employed or immediately laid off.

The total labor force in the economy is assumed to be constant over time and normalized to one. Total time endowment is also normalized to one with agents dividing up their available time between working and enjoying leisure. The decision making of consumer-workers and firms in general equilibrium can be summarized by the following representative agent's welfare maximization problem. The representative agent orders time paths of consumption services and total hours worked according to the criterion

$$E_z \sum_{t=1}^{\infty} \beta^t U(C_t, H_t), 0 < \beta < 1, \tag{2.4}$$

where

$$H_t = h_0 \int_{R_t}^{\gamma} N_t(x) dx, 0 < h_0 < 1. \tag{2.5}$$

The parameter β denotes the common discount factor, and E_z denotes the expectational operator that takes expectations with respect to the random variable z. The variable H_t represents total hours worked with h_0 denoting the fraction of total time endowment that is spent working. R_t stands for the reservation productivity level above which job-matches are employed, and $N_t(x)$ represents the fraction of the attached work force that is characterized by the idiosyncratic productivity x. Preferences are specified as additively separable between consumption and hours spent working:

$$U(C_t, H_t) = \log(C_t) - BH_t, \quad B > 0. \tag{2.6}$$

As I elaborate in Merz (1997), this utility function of the representative agent is derived by assuming that labor is indivisible, and that all agents in the economy can perfectly insure themselves against unemployment. Note that B measures the marginal disutility of labor.

Aggregate per capita output (Y_t) can either be used for private consumption (C_t), investment into new capital (I_t), or for advertising employment opportunities. The cost of posting a vacancy (V_t) is a constant, a, which is measured in terms of the single output good. The economy's aggregate resource constraint can be expressed as the following inequality:

$$C_t + I_t + \exp(\mu t) a V_t \leq Y_t, \quad \mu \geq 0. \tag{2.7}$$

The parameter μ denotes the common rate at which aggregate per capita output, private consumption, and the capital stock grow in nonstochastic steady state. Thus, the model exhibits balanced growth. Aggregate per capita output at any point in time is the result of a constant-returns-to-scale production function given by

$$Y_t \equiv F(z_t H_t^*, K_t) = \exp[(1-\alpha)\mu t](z_t H_t^*)^{1-\alpha} K_t^{\alpha}, \quad 0 \leq \alpha \leq 1, \tag{2.8}$$

where

$$H_t^* = h_0 \int_{R_t}^{\gamma} \exp(x) N_t(x) \mathrm{d}x. \tag{2.9}$$

The accumulated per capita capital stock K_t and total effective hours H_t^* – the productivity-weighted fraction of the attached work force that lies above the reservation productivity level multiplied by the constant shift length – are the inputs into the aggregate production process. The expression $\exp(\mu t) z_t$ denotes labor-augmenting technological progress. N_t stands for the distribution of the available work force across idiosyncratic productivity levels in period t. The per capita capital stock depreciates at the constant rate δ_K in each period and is increased by any investment undertaken. Thus, it obeys the following law of motion:

$$K_{t+1} = (1-\delta_K) K_t + I_t, \quad 0 \leq \delta_K \leq 1. \tag{2.10}$$

The attached work force at every idiosyncratic productivity level x evolves according to

$$N_{t+1}(x) = (1-\delta_N-\xi) N_t(x) + \xi g_x \int_{-\gamma}^{\gamma} N_t(s) ds + g_x M_t, \quad 0 \leq \delta_N \leq 1. \tag{2.11}$$

The variable M_t represents the number of new job-matches that are formed in time period t. The fraction of the attached work force gathering at productivity level x in every given period is determined by the flow of the work force to and from this level during that period. The exogenous portion of the flow away from productivity level x consists of two parts. The first part is the fraction δ_N of the work force that permanently separates through permanent layoffs and quits into unemployment. The second part is the fraction whose idiosyncratic productivity level changes with probability ξ. The flow of the arriving work force also consists of two parts. It contains the fraction of the total attached work force A_t that changes its idiosyncratic productivity with probability ξ and is re-categorized as level x with probability g_x:

$$A_t = \int_{-\gamma}^{\gamma} N_t(x) \mathrm{d}x. \tag{2.12}$$

It also contains the fraction of all new job-matches that exhibit productivity x with probability g_x. New matches can be thought of as being generated by the following Cobb–Douglas function that uses vacancies posted and permanently unemployed workers as inputs:

$$M_t = \theta_t^{1-\lambda}(1 - A_t), \quad 0 \leq \lambda \leq 1, \tag{2.13}$$

where $\theta_t = V_t/(1 - A_t)$ measures the tightness of the labor market. The matching technology M_t implies an endogenous probability for the transition from permanent unemployment to employment, m_t^{NA}, and a likelihood for the transition from an unfilled to a filled job-vacancy, m_t^V, that each depend on the tightness of the labor market:

$$m_t^{NA} = \frac{M_t}{1 - A_t} = \theta_t^{1-\lambda}, \qquad m_t^V = \frac{M_t}{V_t} = \theta_t^{-\lambda}. \tag{2.14}$$

Furthermore, it implies that these transition probabilities decrease with an increasing degree of congestion, and vice versa. Note that, when integrating over all x, Eq. (2.10) implies the law of motion for the total attached work force in the economy:

$$A_{t+1} = (1 - \delta_N)A_t + M_t. \tag{2.15}$$

The social planning problem consists of the planner choosing contingency plans for $\{C_t, V_t, R_t, K_{t+1}, N_{t+1}: t \geq 1\}$ at time 1 in order to maximize the objective function (2.4) subject to (2.1)–(2.3), (2.7)–(2.13), K_0, N_0 and z_0. The social planner is assumed to make period t decisions based on all information available at time t. The timing is such that at the beginning of each period the planner inherits as state variables a capital stock, a distribution of the available work force across different labor productivity levels, as well as the previously realized technology shock. When an aggregate technology shock occurs, the planner decides upon the reservation productivity level that separates the available work force into the fractions employed and temporarily laid off. Employment together with the existing capital stock produce aggregate output during the period. The planner also decides upon the level of investment in new capital, and determines the level of new employment opportunities. These posted vacancies determine the rate at which new job-matches are formed. Together with the recategorization of the work force that takes place as a consequence of shocks to idiosyncratic productivity occurring during the period, these variables determine the distribution of the work force across productivity levels at the beginning of next period. Similarly, new investment adds to the existing capital stock, and, together with the fraction of the work force that will be employed, determines the level of output in the following period.

Since the model exhibits balanced growth, all nonstationary variables have to be detrended in order for the model to be solvable by linear quadratic approximation around the nonstochastic steady state. For that purpose, the detrended versions of the respective variables are defined as follows:

$$\tilde{K}_{t+1} = \frac{K_{t+1}}{\exp(\mu t)}, \qquad \tilde{C}_t = \frac{C_t}{\exp(\mu t)}, \qquad \tilde{I}_t = \frac{I_t}{\exp(\mu t)}. \tag{2.16}$$

Assuming nonsatiation, the aggregate resource constraint is binding, and the social welfare problem that includes only stationary variables can be formulated as the following dynamic programming problem. For the sake of clarity, I delete time subscripts and let primes indicate a variable's future value.

$$W(\Omega) = \max_{\{\tilde{I},\, V,\, R\}} [\log(\tilde{C}) - Bh_0 \int_R^{\gamma} N(x)\mathrm{d}x + \beta E_z W(\Omega')|\Omega)],$$

subject to

$$\tilde{C} = \exp(-\alpha\mu)[zh_0 \int_R^{\gamma} \exp(x)N(x)\mathrm{d}x]^{1-\alpha}\tilde{K}^{\alpha} - \tilde{I} - aV,$$

$$\tilde{K}' = (1-\delta_K^*)\tilde{K} + \tilde{I},$$

$$N'(x) = (1-\delta_N-\xi)N(x) + \xi g_x \int_{-\gamma}^{\gamma} N(s)\mathrm{d}s + g_x M \;\forall x,$$

$$z' = \rho z + \varepsilon',$$

where Ω denotes the current aggregate state that consists of the exogenous state variable z, and the endogenous state variables N and the detrended version of K:

$$\Omega = \{z, N, \tilde{K}\}.$$

Furthermore $\delta_K^* = 1-(1-\delta_K)\exp(-\mu)$. The corresponding Euler conditions for an interior solution shed light on the planner's allocation decision as well as on the procedure used to solve the model:

$$\text{(P1):}\quad U_{\tilde{C}} = \beta E_z\{U_{\tilde{C}'}[F_{\tilde{K}'} + (1-\delta_K^*)]|\Omega\},$$

$$\text{(P2):}\quad a = (1-\lambda)m^V\{\beta\int_{-\gamma}^{\gamma} E_z[W_{N'(x)}|\Omega]g_x\mathrm{d}x\}/U_{\tilde{C}},$$

$$\text{(P3):}\quad F_{N(x)|x=R}U_{\tilde{C}} = B,$$

$$\begin{aligned}\text{(P4):}\quad W_{N(x)} = {} & 1\cdot\{R \le x\}\{U_{\tilde{C}}F_{N(x)} - B\} + \beta\{E_z[W_{N'(x)}|\Omega](1-\delta_N-\xi)\} \\ & + \beta\{\xi\int_{-\gamma}^{\gamma} E_z[W_{N'(s)}|\Omega]g_s\mathrm{d}s \\ & - \int_{-\gamma}^{\gamma} E_z[W_{N'(s)}M_{NA'}|\Omega]g_s\mathrm{d}s\}\;\forall x,\end{aligned}$$

where U_C denotes the derivative of function U with respect to variable C. The remaining notation is analogous.

(P1) gives the standard condition for the optimal intertemporal allocation of consumption. (P2) and (P3) denote the optimality conditions for creating new job-matches and laying off existing ones, respectively. According to (P2), job-matches are created until the advertising cost of posting an additional vacancy just offsets the discounted expected future payoff from such a vacancy, measured in terms of the marginal utility of consumption. Assuming that the planner focuses on an interior solution with $N(R) > 0$, (P3) determines the optimal reservation productivity level. The planner sets the level so that the marginal product of the job-match employed at the reservation level covers the opportunity cost of employment, B. (P4) is an asset pricing equation. It constrains the social value of a match to equal its marginal product net of the opportunity cost of employment that is adjusted for its discounted social value if it survives another period, and also for the discounted social value foregone due to new job-matches that could not be created.

Hence, the planner intertemporally allocates the different fractions of the work force and the capital stock in order to achieve dynamic optimality. Dynamic optimality as well as feasibility require that the corresponding costates satisfy a standard transversality condition according to which the present discounted value of each of these prices in period t, evaluated using period t market prices, tends to zero as t tends to infinity. These transversality conditions are imposed when solving the model. I provide an outline of the procedure that I use to solve the model in Appendix C.

3. Flows into and flows out of unemployment

A key characteristic of all data on labor turnover is that flows into and flows out of unemployment occur simultaneously and at all stages of the business cycle. During the same time period some workers move from the state of employment to the state of unemployment, while others move in the opposite direction. This phenomenon is captured in my model in which job-matches exhibit different levels of idiosyncratic productivity. In any given period, the unemployment rate increases because some job-matches permanently separate, or are temporarily laid off. It decreases because other matches are recalled, or newly created. Thus, the change in the unemployment rate equals the difference between the flows into and the flows out of unemployment. This net change as well as its components can be derived from the basic structure of my model.

By definition, the unemployment rate u equals the fraction of the total labor force that is not employed. Since I assume the labor force to be constant and equal to one, it follows that, in a given period, u is defined as

$$u = 1 - E, \tag{3.1}$$

where E denotes total employment. The change in the unemployment rate between two consecutive periods is equal to

$$\Delta u = u' - u = E - E'. \tag{3.2}$$

When integrating Eq. (2.10) over the range of job-matches employed in the future, the law of motion for aggregate employment follows:

$$E' = \int_{R'}^{\gamma} N'(x)\mathrm{d}x = (1 - \delta_N - \xi)\int_{R'}^{\gamma} N(x)\mathrm{d}x + \xi\int_{R'}^{\gamma} g_x\mathrm{d}xA + \int_{R'}^{\gamma} g_x\mathrm{d}xM. \tag{3.3}$$

It can be used to express Eq. (3.2) as

$$\Delta u = E - \xi\int_{R'}^{\gamma} g_x\mathrm{d}xE - (1 - \delta_N - \xi)\int_{R'}^{\gamma} N(x)\mathrm{d}x - \xi\int_{R'}^{\gamma} g_x\mathrm{d}xTL + \int_{-\gamma}^{R'} g_x\mathrm{d}xM - M. \tag{3.4}$$

When using the fact that

$$\int_{R'}^{\gamma} N(x)\mathrm{d}x = \begin{cases} \int_{R'}^{R} N(x)\mathrm{d}x + E & \text{if } R' < R, \\ E - \int_{R}^{R'} N(x)\mathrm{d}x & \text{if } R' \geq R, \end{cases} \tag{3.5}$$

and after some rearranging the change in the unemployment rate, Δu, can be written as

$$\Delta u = \delta_N E + \xi\int_{-\gamma}^{R'} g_x\mathrm{d}xE - \xi\int_{R'}^{\gamma} g_x\mathrm{d}xTL + \int_{-\gamma}^{R'} g_x\mathrm{d}xM - M + (1 - \delta_N - \xi)[1\cdot\{R' < R\}\{-\int_{R'}^{R} N(x)\mathrm{d}x\} + 1\cdot\{R' \geq R\}\{\int_{R}^{R'} N(x)\mathrm{d}x\}]. \tag{3.6}$$

I define the different parts of worker flows as follows:

$$
\begin{aligned}
\textit{temporary layoffs} &= 1\cdot\{R' \geq R\}\{(1-\delta_N-\xi)\int_R^{R'} N(x)\mathrm{d}x\} + \xi\int_{-\gamma}^{R'} g_x \mathrm{d}x E \\
\textit{other inflows} &= \delta_N E \\
\textit{recalls} &= 1\cdot\{R' < R\}\{(1-\delta_N-\xi)\int_{R'}^{R} N(x)\mathrm{d}x\} + \xi\int_{R'}^{\gamma} g_x \mathrm{d}x TL \\
\textit{active new hires} &= \int_{R'}^{\gamma} g_x \mathrm{d}x M, \qquad (3.7)
\end{aligned}
$$

where *other inflows* capture the sum of permanent layoffs and quits into unemployment. I can use these definitions to introduce flows into unemployment, *in*, and flows out of unemployment, *out*, and the change in the unemployment rate, Δu, as follows:

$$
\begin{aligned}
\textit{in} &= \textit{temporary layoffs} + \textit{other inflows} \\
\textit{out} &= \textit{active new hires} + \textit{recalls} \qquad (3.8) \\
\Delta u &= \textit{in} - \textit{out}.
\end{aligned}
$$

According to Eqs. (3.7) and (3.8), flows into unemployment consist of temporary layoffs and a constant fraction δ_N of total employment. This constant fraction of employment captures permanent layoffs and quits into unemployment. Employed job-matches are temporarily laid off for two different reasons. They are laid off if they receive an idiosyncratic productivity shock that pushes them below next period's reservation productivity level. This event occurs with probability $\xi G(R')$. They are also laid off if their given idiosyncratic productivity is less than next period's increased cutoff rule. Outflows from unemployment consist of the sum of recalls and new hires that are employed. Temporarily laid off job-matches are recalled if they receive an idiosyncratic shock that pushes them above next period's reservation productivity level. This event occurs with probability $\xi[1-G(R')]$. They are also recalled if their given productivity level lies above next period's decreased cutoff rule. New job-matches whose productivity exceeds next period's cutoff rule, are also a part of flows from unemployment to employment.

Having constructed unemployment flows from the basic structure of my model, I can simulate the cyclical behavior of these variables once I have determined plausible model parameters.

4. Unemployment incidence and unemployment duration in steady state

Various relations between a selected set of stock and flow variables of the labor market can be derived from Eq. (3.7). They prove to be useful in defining and measuring unemployment incidence and unemployment duration, and in parameterizing the model. After some rearranging this equation yields expressions for unemployment incidence, Λ, where

$$\Lambda \equiv \frac{in}{E} = \delta_N + G(R)(\xi + M) \tag{4.1}$$

and for unemployment duration, D:

$$\frac{1}{D} \equiv \frac{out}{u} = \xi[1 - G(R)]\frac{TL}{u} + m^{NA}\frac{1-A}{u}. \tag{4.2}$$

The matching probability for permanently separated workers, m^{NA}, is the one that is defined in Eq. (2.14).

Unemployment incidence equals the sum of the probability of being permanently separated or temporarily laid off. Unemployment duration is inversely related to the probability of moving from unemployment to employment. This probability corresponds to the weighted average of the probability of being recalled when temporarily laid off, and the probability of being rematched when permanently separated. The weights equal the fraction of the unemployed that are temporarily laid off, and the fraction that are permanently separated, respectively. In a steady state, flows into unemployment equal flows out of unemployment. Therefore, dividing Eq. (4.1) by Eq. (4.2), it follows that

$$\frac{u}{E} = \frac{\delta_N + G(R)\left(\xi + \dfrac{M}{E}\right)}{\xi[1 - G(R)]\dfrac{TL}{u} + m^{NA}\dfrac{1-A}{u}} \equiv D \cdot \Lambda. \tag{4.3}$$

Hence, the ratio between the stock of unemployment and the stock of employment depends on the joint importance of unemployment duration and incidence. Furthermore, the same ratio is compatible with many possible combinations of unemployment duration and incidence. A given ratio can be determined by many workers becoming unemployed for a rather brief spell, or by few workers becoming unemployed for a long time period. Since there is data available on unemployment duration and unemployment incidence for the US economy, these expressions can be used for determining some parameters of the model.

5. Model calibration[3]

In order to derive results for the hypothetical economy, specific values need to be assigned to the parameters α, β, γ, δ_K, δ_N, λ, μ, ξ, ρ, σ_ε, a and B. Following the

[3] I thank Richard Rogerson for pointing out inconsistencies in this section in an earlier version of this paper.

Table 2
Parameter values used for calibration

Symbol	Value	Symbol	Value
α	0.36	μ	0.004
β	0.99	ξ	0.60
γ	0.97	a	1.00
δ_K	0.022	B	0.332
δ_N	0.064	ρ	0.95
λ	0.50	σ_ε	0.0071

Notes: The parameter α denotes output's elasticity with respect to the capital stock, β the discount rate, γ the highest possible level of idiosyncratic productivity, δ_K the capital stock's depreciation rate, δ_N the transition rate from employment to permanent unemployment, λ the elasticity of job-matches with respect to permanent layoffs, μ the common growth rate, ξ the probability with which the idiosyncratic productivity changes, a the per vacancy advertising cost, B the disutility of labor, ρ the autocorrelation coefficient of the aggregate technology shock, and σ_ε the standard deviation of its innovations.

work of Kydland and Prescott (1982), it has become a standard practice in the real business cycle literature to choose parameter values that are based on growth observations and studies using microeconomic data. To make my results comparable to those of other studies, I follow convention. The parameters I use for calibrating the model are summarized in Table 2.

Given the Cobb–Douglas production function, the parameter α represents the elasticity of aggregate output with respect to physical capital. Using US time series data Kydland and Prescott (1982) found this parameter to be approximately equal to 0.36. It equals the capital share of GNP. However, as wages do not correspond to the marginal product of labor in my model, $(1-\alpha)$ is not equal to the labor share of GNP. It is equal to the sum of the labor share of total income and the return to investing in job-search. The discount factor β, common to all individuals, is set equal to 0.99, implying an annual rate of interest of four percent in nonstochastic steady state. The quarterly rate of depreciation of the capital stock δ_k is set equal to 0.022, implying a steady state ratio of private consumption to aggregate output equal to 0.72, and a ratio of capital investment to aggregate output equal to 0.279. Together with $\mu = 0.004$ – which amounts to assuming an overall annual growth rate of 1.6% – this corresponds to an effective annual depreciation rate δ_K^* of ten percent.

I use the data on unemployment flows and stocks constructed as described in Appendix A in order to calculate first moments of some key labor market variables. The unemployment rate for the time period extending from the second quarter of 1977 to the fourth quarter of 1996 equals 6.7%. Flows to and from unemployment expressed as a fraction of the total labor force each equal 7.46%

per quarter over the same time period. As can be seen from Eq. (4.1), this information implies an average quarterly probability of becoming unemployed, Λ, equal to 8%. This value states that, on average, a person is employed for 3.1 years before becoming unemployed. Using the definitional link in Eq. (4.2), the information on the rate and incidence of unemployment implies an average duration of unemployment equal to 0.9 quarters, or close to eleven weeks. Total layoffs constitute 48% of flows into unemployment. Forty percent of all layoffs are temporary. Hence, about 19% of all flows to unemployment are temporary layoffs, and 29% are due to permanent layoffs. The remaining fraction of inflows consists of quits into unemployment, and entrants to the labor force. Similarly, 83% of flows from unemployment are due to new match creations. Recalls explain the remaining 17%.

I normalize the per unit advertising cost, a, which firms pay for posting a vacancy by setting it equal to one. I set δ_N equal to 0.064 so that on average 81% of all inflows are due to permanent separations and entrants to the labor force. Taking all other parameters as given, I simultaneously determine the parameters B, λ and ξ so that the model matches the average rate and incidence of unemployment, and also the average ratio of recalls to outflows. The relative volatility of employment to aggregate output equals 0.66. It is necessary to match a second moment of the labor market, since the parameter γ traces out the length of the interval of productivity levels. This interval affects the impact a change in the reservation productivity has on employment. For any given change, this impact varies inversely with the length of the interval. The following parameter values satisfy all of these restrictions simultaneously: $B = 0.332$, $\lambda = 0.50$, $\xi = 0.60$ and $\gamma = 0.97$. They imply an autocorrelation coefficient for the idiosyncratic productivity process of 0.40.

Finally, I parameterize the law of motion for the technology shock by setting ρ equal to 0.95. I set σ_ε equal to 0.0071 – a value commonly found in the RBC-literature. This allows me to compare my model's predictions on output volatility, for example, to those of existing studies.

6. Simulations

My goal is to evaluate the performance of a one-sector stochastic growth model augmented by matching frictions in the labor market and match-specific shocks in generating the observed volatility, persistence, cyclicality and dynamic comovement of unemployment flows and the unemployment rate. I use a baseline version to generate artificial series of the unemployment rate and of the various components of unemployment flows. I compute second moments of those series and contrast them to their empirical counterpart. I also perform a sensitivity analysis by varying single parameters.

6.1. Simulation procedure

To obtain a large number of samples of artificially generated time series I simulate the model five hundred times. Each sample has the same number of periods as the time series used in this analysis. Their statistical properties can be compared to the ones computed for the respective US data. Unemployment flows and the unemployment rate are constructed from data on unemployment duration by cause. These data have been available since 1977 from the Bureau of Labor Statistics. The source of all remaining series and the construction of the data is explained in detail in Appendix A.

All time series are logged, and deviations from trend are computed with the help of the Hodrick–Prescott filter. The cyclical properties of the time series of both the US economy and the model economy are summarized by a set of relative standard deviations and contemporaneous correlation coefficients. The time period covered ranges from the second quarter of 1977 to the fourth quarter of 1996.

6.2. Simulation results

I first simulate the model with the baseline parameters reported in Table 2. I summarize the simulation results by reporting selected second moments in Tables 3–5. I supplement these results by impulse response functions which I present in Fig. 2. These functions enhance our understanding of the mechanisms generating the results. I also perform a sensitivity analysis by varying the parameter ξ – the probability with which the match-specific productivity changes. I report my findings in Table 6.

6.2.1. The benchmark case

The simulation results convey a clear message. When the one-sector stochastic growth model is augmented by trade frictions in the labor market and match-specific idiosyncratic shocks, it generates unemployment flows whose dynamic characteristics closely resemble those of their empirical counterparts. The model captures temporary layoffs and permanent separations as components of inflows, and recalls and new match creations as parts of outflows. The simulation results can identify the role that each subflow plays in generating the observed volatility, persistence, and cyclicality of total unemployment flows and of the unemployment rate. The results correctly point to inflows as the main driving force underlying the dynamics of the unemployment rate. The model also mimics the dynamic behavior of private consumption and investment in physical capital, but cannot generate the observed volatility of output.

I measure volatility by a variable's standard deviation relative to output, and persistence by its degree of autocorrelation. The model mimics the empirical

Table 3
Selected second moments from US and artificial economy

Statistic	US data 1977:2–1996:4	Model economy	Statistic	US data 1977:2–1996:4	Model economy
I. Commonly reported values					
σ_C/σ_Y	0.43	0.32 (0.017)	σ_E/σ_Y	0.66	0.66 (0.006)
σ_I/σ_Y	3.07	2.81 (0.024)	σ_u/σ_Y	6.31	4.83 (0.050)
σ_Y	1.56	0.64 (0.001)	σ_V/σ_Y	8.82	2.02 (0.131)
II. Unemployment flows					
σ_{in}/σ_Y	4.31	2.50 (0.105)	σ_{out}/σ_Y	3.45	2.36 (0.116)
σ_{tl}/σ_Y	7.66	14.51 (0.942)	σ_{rc}/σ_Y	6.70	14.74 (0.521)
σ_{pl}/σ_Y	3.45	0.66 (0.007)	σ_m/σ_Y	3.29	1.12 (0.139)

Notes: See Table 1. C denotes private consumption, Y output, and I capital investment. All of these variables are expressed in per capita terms. E denotes employment, u unemployment rate, and V job-vacancies. σ_x/σ_y represents the ratio between the standard deviation of variable x and the standard deviation of variable y. Appendix A contains information regarding the source and construction of the data series used here. All statistics are computed after detrending the logarithm of the data using the Hodrick–Prescott filter. The standard deviations are sample means of statistics computed for each of 500 simulations. Each simulation consists of 79 periods. The numbers in parentheses are sample standard deviations of these statistics.

observation that the unemployment rate and unemployment flows are highly volatile and persistent. Moreover, it correctly predicts the unemployment rate to be more volatile than both flow variables, and inflows to fluctuate more strongly than outflows. The model also generates the fact that the unemployment rate is more persistent than outflows which, in turn, are more persistent than inflows. Even though the model comes close to matching the empirical persistence of the unemployment rate, it predicts unemployment flows to be less persistent than their empirical counterparts.

Tables 3 and 4 nicely illustrate the role that different subflows play in generating the observed dynamics. Temporary layoffs are more volatile and less persistent than permanent layoffs. Recalls fluctuate more strongly than new

Table 4
Autocorrelations for US and artificial economy 1977:2–1996:4

	Statistic	1	2	3
I. Unemployment inflows				
US data	$\rho(in_t, in_{t-\tau})$	0.793	0.541	0.341
	$\rho(tl_t, tl_{t-\tau})$	0.694	0.340	0.146
	$\rho(pl_t, pl_{t-\tau})$	0.704	0.555	0.460
Model economy	$\rho(in_t, in_{t-\tau})$	0.456	0.282	0.141
	$\rho(tl_t, tl_{t-\tau})$	0.535	0.340	0.167
	$\rho(pl_t, pl_{t-\tau})$	0.740	0.437	0.232
II. Unemployment outflows				
US data	$\rho(out_t, out_{t-\tau})$	0.803	0.618	0.450
	$\rho(rc_t, rc_{t-\tau})$	0.675	0.395	0.155
	$\rho(m_t, m_{t-\tau})$	0.767	0.593	0.449
Model economy	$\rho(out_t, out_{t-\tau})$	0.628	0.281	0.167
	$\rho(rc_t, rc_{t-\tau})$	0.468	0.304	0.148
	$\rho(m_t, m_{t-\tau})$	− 0.382	0.065	− 0.062
III. Unemployment rate				
US data	$\rho(u_t, u_{t-\tau})$	0.904	0.724	0.518
Model economy	$\rho(u_t, u_{t-\tau})$	0.740	0.437	0.232

Notes: See Tables 1 and 3.

hires. Hence, the model correctly predicts that temporary layoffs and recalls are crucial for generating the observed amount of volatility. It identifies permanent layoffs and new hires as important sources for persistence. Permanent layoffs are as persistent as their empirical counterpart. But the model counterfactually predicts new hires to be little persistent and negatively autocorrelated. Advertising costs are the main culprit. When a technology shock hits, employment adjustment via new hires lasts for a brief period only. Further adjustment occurs through ongoing layoffs and recalls which are free. Consequently, new hires are less persistent than recalls and also less persistent than what we observe in the data. This phenomenon translates into a lack of persistence in outflows.

I measure a variable's cyclicality by its comovement with the unemployment rate. The model correctly predicts inflows and outflows to behave countercyclically. It generates a positive contemporaneous correlation between inflows and outflows, but the predicted correlation is less than what we find in the data. The model performs remarkably well in generating the dynamic cross correlations

Table 5
Dynamic correlations for US and artificial economy1977:2–1996:1

	Statistic	τ = −2	−1	0	1	2
I. *Unemployment inflows and unemployment rate*						
US data	$\rho(in_t, u_{t-\tau})$	0.796	0.876	0.825	0.585	0.348
	$\rho(tl_t, u_{t-\tau})$	0.678	0.675	0.541	0.221	−0.076
	$\rho(pl_t, u_{t-\tau})$	0.759	0.856	0.866	0.678	0.495
Model data	$\rho(in_t, u_{t-\tau})$	0.743	0.919	0.450	0.253	0.090
	$\rho(tl_t, u_{t-\tau})$	0.740	0.941	0.532	0.317	0.131
	$\rho(pl_t, u_{t-\tau})$	−0.431	−0.738	−1.0	−0.738	−0.431
II. *Unemployment outflows and unemployment rate*						
US data	$\rho(out_t, u_{t-\tau})$	0.603	0.758	0.853	0.848	0.737
	$\rho(rc_t, u_{t-\tau})$	0.669	0.724	0.684	0.513	0.237
	$\rho(m_t, u_{t-\tau})$	0.386	0.542	0.678	0.763	0.773
Model data	$\rho(out_t, u_{t-\tau})$	0.252	0.514	0.935	0.817	0.467
	$\rho(rc_t, u_{t-\tau})$	0.313	0.606	0.960	0.625	0.392
	$\rho(m_t, u_{t-\tau})$	−0.213	−0.342	−0.243	0.371	0.126
III. *Unemployment inflows and outflows*						
US data	$\rho(in_t, out_{t-\tau})$	0.761	0.806	0.765	0.551	0.292
Model data	$\rho(in_t, out_{t-\tau})$	0.829	0.776	0.228	0.137	0.014

Notes: See Tables 1 and 3.

between the unemployment rate and unemployment flows. It correctly predicts inflows to lead the unemployment rate by one quarter over the business cycle and points to temporary layoffs as the major source of this phenomenon. Similar to what we find in the data, temporary layoffs lead the unemployment rate. But given the counterfactual negative correlation between permanent layoffs and the unemployment rate, the model is likely to overemphasize the role played by temporary layoffs in adjusting employment. If permanent layoffs were also explicitly modeled, temporary layoffs would have to bear less of the burden of adjusting employment in reaction to aggregate shocks. This modification can be expected to enhance the countercyclicality of inflows, and to take the model even closer to the data. In its current version, the model replicates the observation that the unemployment rate slightly leads outflows over the business cycle. In the data, this timing relation is due to recalls leading the unemployment rate by a quarter and new hires following it by two quarters. According to my model, new hires follow the unemployment rate by one quarter and recalls contemporaneously change with the unemployment rate.

The key to understanding these model predictions lies in the cyclical behavior of the reservation productivity and job-vacancies, and in the timing of the resulting unemployment flows. The reservation productivity is countercyclical. It affects unemployment flows primarily in the period in which it changes. Vacancies are procyclical. They affect the creation of new job-matches which can be productive one period after their creation. As Fig. 2 illustrates, when a positive technology shock hits the economy, the reservation productivity drops immediately and the number of vacancies posted increases. The immediate decline in the cutoff rule increases employment and reduces the stock of temporary layoffs. Flows into unemployment decline because a strong decrease in the flow of temporary layoffs outweighs an increase of permanent layoffs due to a larger stock of employment. Fewer job-matches are temporarily laid off, since fewer existing matches receive an idiosyncratic shock that reduces their productivity below the decreased reservation productivity. Also, fewer of the newly created matches are immediately dissolved. Flows out of unemployment decline upon impact of the positive shock because the strong decline in recalls outweighs the increase in new matches. The stock of temporarily laid off matches is sufficiently reduced so that fewer are exposed to idiosyncratic shocks changing their productivity above the decreased reservation level. This effect dominates an increase in outflows due to an increase in newly created matches.

The model replicates the fact that inflows lead the unemployment rate and outflows by one period over the business cycle. The model also correctly predicts the unemployment rate to slightly lead outflows over the cycle. Fig. 2 nicely illustrates the driving forces behind this result. The initially negative change in the unemployment rate signals that the decline of inflows outweighs the decline of outflows upon impact of the shock, leading to a decline in the unemployment rate. It is in that sense that inflows in my model are the main driving force behind an initial change in the unemployment rate. Outflows are the product of the hazard to move from unemployment to employment, out/u, and the unemployment rate. The outflow hazard is procyclical mainly because of procyclical new hires. Since the decline in the unemployment rate is stronger than the increase in the outflow hazard rate, outflows decline even further. Outflows start increasing with an increase in the unemployment rate. There is an alternative interpretation of the same observations. When a positive technology shock occurs, unemployment declines because both, the incidence and duration of unemployment decline.

The model predicts per capita output to be 2.5 times less volatile than its empirical counterpart. This lack of volatility is striking and deserves special attention. The key to the issue lies in the fact that, with heterogeneity, an

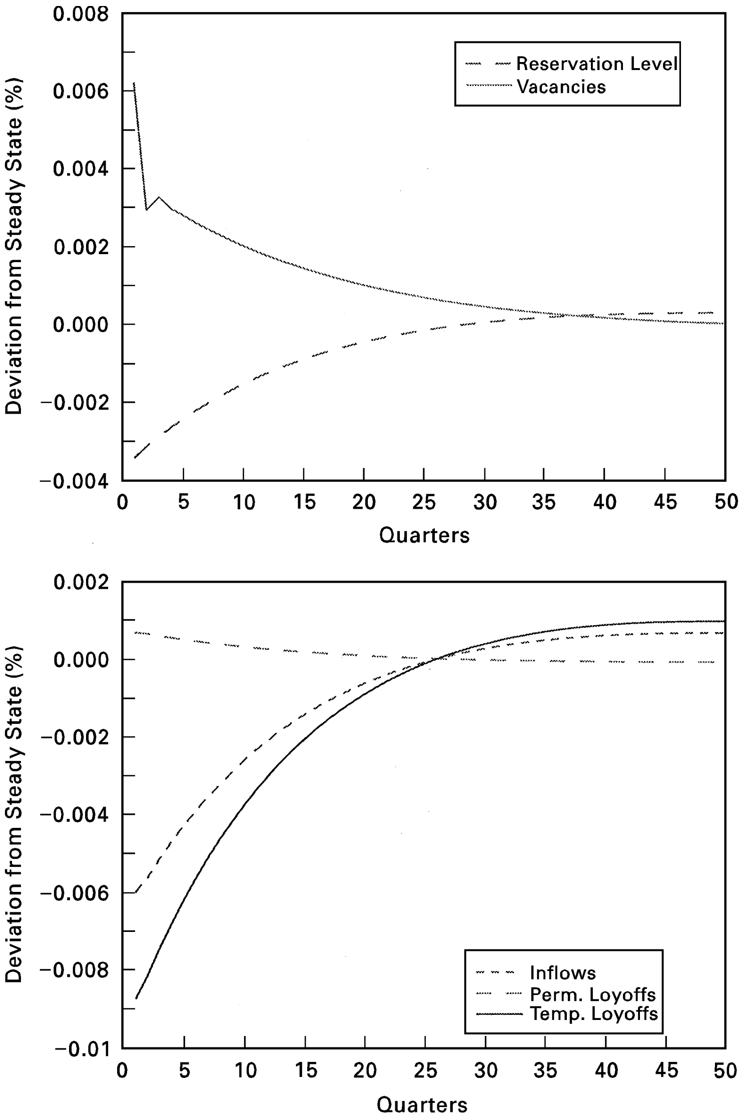

Fig. 2. Impulse response functions.
Notes: The impulse response functions depict responses to a one-standard-deviation positive shock to technology.

important part of employment adjustment occurs at the lower end of the productivity distribution, so that employment changes are small measured in efficiency units. Highly volatile endogenous layoffs and recalls are important in adjusting employment in reaction to aggregate shocks. In a boom,

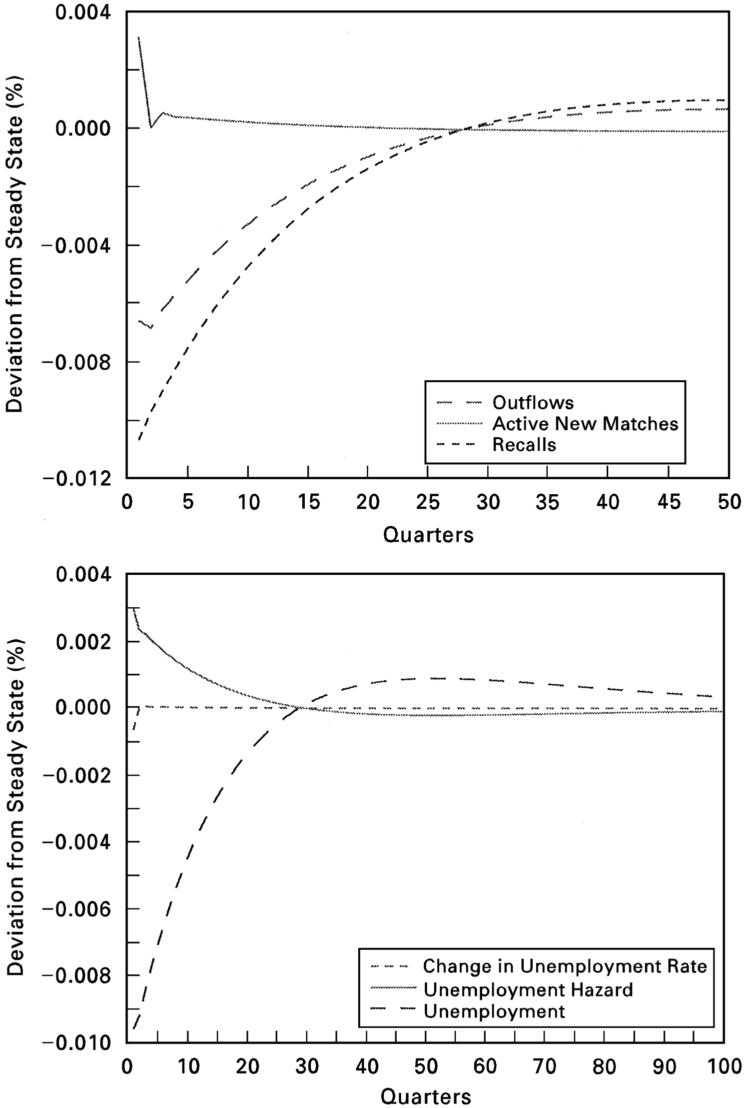

Fig. 2. (continued).

low-productivity matches are recalled, and in a recession, they are the first to be laid off. Since matches enter labor input weighted by their respective labor productivity, a change in the employment status of low-productivity matches has only a small impact on total output. One way to increase output variability would be to increase the relative amount of layoffs and hires of high-productivity matches that occur in recessions and booms. Reducing the degree of heterogeneity by reducing the size of the interval from which idiosyncratic productivity shocks are drawn can only partially alleviate the problem. Such a reduction increases the relative weight of low-productivity matches in the production process, leading to increased output volatility for a given change in the reservation productivity level. But a reduction in the size of the interval also increases the mass of matches that a change in the reservation productivity level affects, thereby increasing the volatility of temporary layoffs, recalls, and the unemployment rate relative to output. Hence, reducing the amount of heterogeneity tends to increase output volatility, but it also drastically increases the relative volatility of unemployment flows and the unemployment rate. I illustrate this point in Table 6 which reports results from a sensitivity analysis.

6.2.2. Sensitivity analysis

I have performed a sensitivity analysis by *ceteris paribus* varying the parameters ξ – the probability of a job-match changing its idiosyncratic productivity-,

Table 6
Sensitivity analysis

I. Autocorrelation

Statistic	1	2	3
$\xi = 0.80$			
$\rho(in_t, in_{t-\tau})$	0.569	0.341	0.164
$\rho(out_t, out_{t-\tau})$	0.716	0.376	0.195
$\rho(u_t, u_{t-\tau})$	0.734	0.422	0.213
$\xi = 0.40$			
$\rho(in_t, in_{t-\tau})$	0.299	0.199	0.077
$\rho(out_t, out_{t-\tau})$	0.497	0.139	0.097
$\rho(u_t, u_{t-\tau})$	0.761	0.468	0.247

II. Volatility and contemporaneous cross correlation

Statistic	σ_{in}/σ_Y	σ_{out}/σ_Y	$\rho(in, out)$	σ_u/σ_Y	σ_Y
$\xi = 0.80$	2.89	2.71	0.461	4.82	0.63
$\xi = 0.40$	2.11	2.02	– 0.080	4.83	0.66
$\gamma = 0.80$, $B = 0.403$	2.92	2.77	0.250	5.40	0.66
$\gamma = 1.10$, $B = 0.278$	2.23	2.11	0.235	4.51	0.63

Notes: See Tables 1 and 3. When varying ξ, I hold all remaining parameters constant at their benchmark level. When varying γ, I adjust the parameter B so that the steady state unemployment rate remains constant at 6.7%.

and the parameter γ which governs the size of the interval from which idiosyncratic productivity shocks are drawn. I have done so in an attempt to shed some more light on the driving forces behind the simulation results. The exercise also aims to enhance the understanding of the link between persistence and volatility of unemployment flows and the unemployment rate, and also of output volatility. I report the simulation results in Table 6.

With an increase in ξ the volatility of inflows and outflows increases. So does their degree of persistence and their contemporaneous correlation. An increase in the parameter ξ raises the total amount of labor turnover. In every period more job-matches receive idiosyncratic shocks that move their match-specific productivity beyond the cutoff rule, leading to a rise in recalls and temporary layoffs. The rise in total labor turnover explains why the persistence of inflows and outflows rises while the persistence of the unemployment rate is largely unaffected. With an increase in ξ the relative importance of recalls for flows out of unemployment, and of temporary layoffs for flows into unemployment increases. Since recalls and temporary layoffs are strongly positively correlated, a rise in ξ translates into an increased positive contemporaneous correlation between inflows and outflows.

7. Conclusions

I have studied the implications of incorporating matching frictions in the labor market and match-specific productivity shocks into an otherwise standard one-sector stochastic growth model on the dynamic behavior of the unemployment rate and unemployment flows. The model structure is rich enough to host a variety of endogenously generated flows to and from unemployment such as new hires, temporary layoffs and recalls. The model replicates the fact that inflows – rather than outflows – are the key driving force of unemployment dynamics. It correctly identifies temporary layoffs and recalls as important sources of volatility, and permanent layoffs and new hires as major sources of persistence. My results indicate important extensions and potentially fruitful avenues for future research.

Incorporating permanent layoffs – in addition to temporary layoffs – as another endogenously determined part of labor demand seems to be the natural next step to take in extending the model. Permanent layoffs can be expected to be countercyclical, thereby reinforcing the countercyclicality of flows into unemployment, and possibly also its degree of variability. This would move the simulation results even closer to the data. However, introducing permanent layoffs as an endogenous variable is a nontrivial extension. Den Haan et al. (1997) recently have taken an important step towards resolving some of the technical challenges that such an exercise involves. Furthermore, the model can be used as an important building block for a more comprehensive framework that studies the forces underlying the dynamic behavior of all worker flows by relaxing the assumption of a constant labor force. One way to do so would be to introduce home production as an alternative to market production, and to allow for worker movement between employment and out of the labor force. Finally, due to the Pareto optimal environment, the analysis presented can be viewed as a benchmark. Allowing for non-optimal wage setting schemes promises to shed further light on the forces underlying the dynamics of unemployment flows and the unemployment rate.

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Appendix A. Data sources and data construction

All data are quarterly real aggregate data of the US for the sample period 1977:2–1996:4. The source for private consumption, investment, output, employment, and vacancies is Fame economics' data bank. Net unemployment flows by cause are constructed from unemployment duration data that originate from the Bureau of Labor Statistics' Current Population Survey. When a series is reported at a monthly frequency, I transform it to quarterly entries by taking simple time-averages.

A.1. Series other than unemployment flows

All series other than unemployment and unemployment flows are constructed from data that are readily available from the Fame economics tape. In what follows, I represent each original series by its Fame label and explain how I compile it in order to obtain the desired series.

[1] GCNQ: Personal consumption expenditures on nondurables (1987 dollars)
[2] GCSQ: Personal consumption expenditures on services (1987 dollars)
[3] GCDQ: Personal consumption expenditures on durables (1987 dollars)

[4] GGEQ: Government purchases of goods and services (1987 dollars)
[5] GIF: Gross private domestic investment, fixed (1987 dollars)
[6] LHEM: Total employment
[7] LHUR: Unemployment rate, all workers 16 +
[8] LHELX: Help-wanted advertising in newspapers to number of unemployed
[9] LHP16: Labor force participation rate, 16 +
[10] P16: Civilian noninstitutional population, total, 16 +

C	Consumption = ([1] + [2] + [4])/[10]
I	Investment = ([3] + [5])/[10]
Y	Output $= C + I$
E	Employment = [6]/[10]
V	Vacancies = [8] × [7] × [9]

A.2. Net unemployment flows by reason for unemployment

Net unemployment flows are derived from an identity that is based on the definition of the unemployment rate. Let u represent the unemployment rate, U the number of individuals unemployed, and L the size of the labor force in a given period. By definition

$$u = \frac{U}{L}. \tag{A.1}$$

As Darby et al. (1986), (p. 615) show, it follows then that

$$\Delta u = \frac{1}{L}\Delta U - \frac{1}{L}\frac{U_{-1}}{L_{-1}}\Delta L, \tag{A.2}$$

and therefore,

$$\Delta u = \frac{1}{L}\Delta U - \frac{\gamma}{1+\gamma}u_{-1}, \tag{A.3}$$

where γ denotes the growth rate of the labor force, and Δx the change in variable x. The change in the number of unemployed individuals, ΔU, equals the number of those who become unemployed less the number of those who leave the state of unemployment during the period. Denoting the former by IN^U, flows into unemployment, and the latter by OUT^U, flows out of unemployment, Eq. (A.3) can be rewritten as follows:

$$\Delta u = \frac{1}{L}(IN^U - OUT^U) - \frac{\gamma}{1+\gamma}u_{-1}. \tag{A.4}$$

After determining the quarterly growth rate of the labor force, γ, this identity is the key to calculating time series on Δu, IN^U and OUT^U.

I use the Bureau of Labor Statistics' data on unemployment duration by reason for unemployment to construct unemployment flows. The total number of unemployed consists of job losers (permanent and temporary layoffs), job leavers, new entrants and reentrants to the labor force. The civilian noninstitutional labor force represents the variable L. Except for the labor force, the series are not adjusted for seasonality. I seasonally adjust the data by first regressing them on 12 dummies. I then apply the ESMOOTH procedure in RATS 4.0 to the residuals from the regression. This procedure smooths the remaining series with the help of parameter estimates that it derives from error-correction models using a simplex method. It is described in more detail in Doan (1992).

I assume that the flow into unemployment is uniformly distributed over time, so the variable IN^U is linearly proportionate to the number of people who have been unemployed for less than five weeks, U^{0-5}. I determine the factor of proportionality by taking a standard month with 30.4 days ($30.4 = 365/12$) and a standard week with 7.02 days ($7.02 = 365/52$) as given. In that case, 86.7% of those who have been unemployed for less than five weeks have become unemployed during the previous month. Note that $0.867 = 30.4/(5 \times 7.02)$. Hence, $IN^U = 3 \times 0.867 \cdot U^{0-5}$ in any given quarter. Given the time series on u, L and IN^U, and also the average value of γ, the flows out of unemployment, OUT^U, can be calculated as a residual from Eq. (A.4). I let *in* denote IN^U/L, and *out* denote OUT^U/L.

Appendix B. A sketch of the labor market in the model economy

The circle in Fig. 3 depicts the constant labor force that is normalized to one. It contains all job-matches that are attached, and either belong to the pool of total employment or to the pool of temporary layoffs, depending on whether the match-specific productivity lies above or below the reservation productivity level. With a fixed probability, each match is dissolved, thereby joining the pool of the unattached work force which equals permanent unemployment.

The diagram on the right-hand side depicts the distribution of the attached work force across all possible idiosyncratic productivity levels. By definition, recalls constitute all job-matches that change their status from being temporarily unemployed to becoming employed. A recall occurs whenever the productivity of a job-match changes from lying below the reservation productivity to lying above it. It occurs if either the reservation productivity level decreases, or the match draws a shock that changes its productivity so that it falls below the reservation productivity, or both events occur at the same time. Temporary layoffs can be explained analogously.

Finally, new job-matches are generated by the number of job-vacancies posted and the size of the pool of permanent unemployment. By assumption,

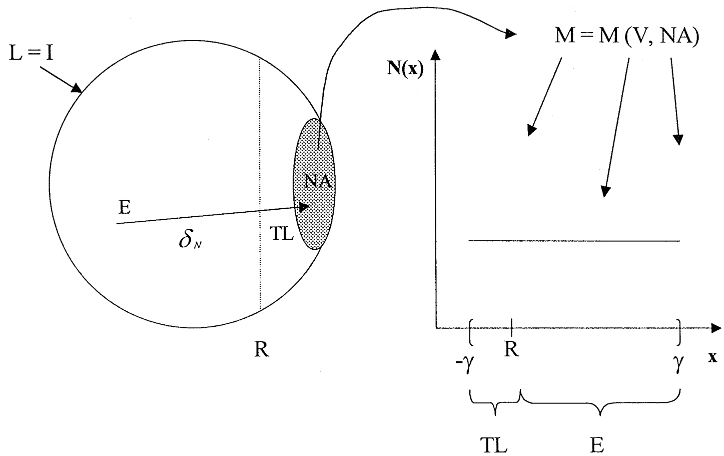

Fig. 3. A sketch of the labor market.

Notation

L	labor force
E	employment
NA	permanent unemployment
TL	temporary unemployment
U	total unemployment, $U = NA + TL$
δ_N	exogenous separation probability
x	idiosyncratic labor productivity
R	reservation productivity level
$N(x)$	fraction of the work force at productivity x
M	new job-matches
V	job-vacancies posted

they are uniformly distributed across all possible productivity levels, that is, they can be temporarily laid off or employed, depending on whether their idiosyncratic productivity falls above or below the reservation level.

Appendix C. Solution procedure for model with heterogenous job-matches

Solving the model means finding the link for the costate and state variables between two consecutive periods. Given the nonlinear nature of the problem, in general, this connection cannot be solved for analytically. However, it can be approximated rather precisely by linearizing the Euler equations of the maximization problem around the nonstochastic steady state and finding a unique solution to the resulting linear system of dynamic equations. This latter method, which is referred to as the state-space approach to linearization, is explained in detail in King et al. (1987). I use an extended version of this procedure to solve this model.

The costates of the model consist of the shadow price W_K that measures capital's marginal contribution to social welfare, and of the continuum of shadow prices $W_{N(x)}$ for each x. The states include the distribution of the attached work force across the different productivity levels x, N, the aggregate capital stock K, as well as the aggregate technology shock z. I approximate the interval $[-\gamma,\gamma]$ by a finite set of T grid points that are equally spaced across the interval. I choose T large enough so that when increasing the number of grid points, the nonstochastic steady state does not change anymore. Thus, the total state space contains $T+1$ costate and $T+2$ state variables. I approximate all integrals in the model numerically by Gaussian quadrature.[4] This approximation produces a finite dimensional linear dynamical system. Although its dimension is much larger than that of a standard problem, its solutions can be found by applying standard linear algebraic techniques.

I express the system of Euler equations as log-linear deviations from stationary steady state and linearize it around steady state with the help of a first-order Taylor series expansion, the only exception being the reservation productivity level R_t which enters as level. I then reduce the system to one in which all decision variables are expressed as linear functions of costates and states of two consecutive periods:

$$A \times Y_{t+1} = B \times Y_t + v_{t+1}, \tag{C.1}$$

where Y represents the $(2\cdot \mathrm{T}+3)\times 1$ vector of stacked costates and states expressed as log-linear deviations from steady state. The $(2\cdot \mathrm{T}+3)\times(2\cdot \mathrm{T}+3)$ matrices A and B contain the corresponding coefficients, and the $(2\cdot \mathrm{T}+3)\times 1$ vector v includes the error terms. Since the matrix A is non-singular, this system of dynamic equations can be rearranged to yield

$$Y_{t+1} = \Pi \times Y_t + A^{-1} \times v_{t+1}, \quad \Pi = A^{-1} \times B. \tag{C.2}$$

In general, there are many Y_t-sequences that satisfy this system with initial conditions given by K_0, N_0 and z_0. This multiplicity of solutions arises because the initial vector Y_0 is undetermined; it contains a $(T+1)$-dimensional degree of freedom, W_K and $W_{N(x)}$ at time period zero. In order to find a singleton solution, the following transversality condition has to be imposed: $\beta^t Y_t \to 0$ as $t \to \infty$ with probability one. Given that the costates are shadow prices that can be considered as prices accompanying the dynamic assets in the model, capital and attached work force, this transversality condition can be interpreted as a non-bubble condition which ensures that these prices do not explode in equilibrium.

[4] For further details on the Gaussian quadrature approximation of integrals see, for example, Press et al. (1992).

This condition also ensures that the error term v can be ignored if and only if there are exactly $(T+1)$ eigenvalues of the matrix Π that lie outside the circle spanned by 1. In this case, there exists a unique solution that satisfies the transversality condition.[5] It is given by

$$Q \times Y_t = Q_a \times Y_{1_t} + Q_b \times Y_{2_t} = 0, \quad t = 0,1,2\ldots, \tag{C.3}$$

where the rows of the $(\mathrm{T}+1) \times (2 \cdot \mathrm{T}+3)$ matrix Q are the left eigenvectors of Π associated with its explosive eigenvalues, and Y_{1t} represents the vector of costates and Y_{2t} the one of states. Hence, the mapping from states into costates at time t is given by

$$Y_{1_t} = \Theta \times Y_{2_t}, \quad \Theta = Q_a^{-1} \times Q_b, \tag{C.4}$$

and the mapping from states at time t into costates and states at $t+1$ is given by

$$Y_{t+1} = \Pi \begin{bmatrix} \Theta \\ I \end{bmatrix} Y_{2_t},$$

where I denotes a $(T+2) \times (T+2)$ identity matrix.

With the help of the linear dynamic systems (C.4) and (C.5), I can compute the log-deviations from steady state of all model variables. These can be translated into levels by taking the anti-log and then multiplying the outcome by the respective steady state value.

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[5] If the number of eigenvalues that lie outside this circle exceeds the number of non-predetermined variables, there is no solution satisfying both Eq. (C.1) and the transversality condition. If the number of those eigenvalues is less than the number of non-predetermined variables, there is an infinity of solutions. See Blanchard and Kahn (1980) for proofs.

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