A portfolio approach to endogenous growth: equilibrium and optimal policy*

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Abstract

This paper develops a portfolio approach to modeling endogenous growth in continuous time that is especially suitable for addressing fiscal and financial issues in policy design. The analysis focuses on the equilibrium relationship between fiscal and financial policy, rates of return and wealth allocation. We analyze two models. The first is based on the Arrow–Romer model with increasing returns and an external effect of capital on labor productivity. The second draws on Barro’s analysis of government spending and endogenous growth. In both models, we study the equilibrium allocation and discuss the optimal fiscal and financial policy.

Keywords: Endogenous growth; Risk; Asset pricing; Optimal taxation

JEL classification: D9; H2; G1

1. Introduction

This paper develops a portfolio approach to endogenous growth models in continuous time that analyzes the equilibrium relationship between fiscal and financial policy, rates of return of financial assets and wealth allocation. We illustrate the features of our approach by analyzing two models. The first is a stochastic version of the Arrow–Romer model with increasing returns and an external effect of capital on labor productivity (as in Romer, 1986). The second is the model of government spending and endogenous growth discussed by Barro (1990).

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Our analytical framework is an intertemporal CAPM in general equilibrium, in which the macroeconomic process is analyzed from the standpoint of wealth allocation. Its main characteristics are as follows. At firm level, the production function is a standard Cobb–Douglas including both labor and capital as arguments; yet, from a social point of view, technology in our model is linear in capital. With either capital (in Section 2) or government spending (in Section 3) exerting an external effect on production, the productivity of labor grows with the outstanding stock of capital.

Output is subject to an instantaneous, economy-wide shock common to all firms, modeled as a Brownian motion with drift. In this respect, our models are related to work on financial issues by Eaton (1981), Gertler and Grinols (1982) and Stulz (1986). In these papers, the returns of all financial assets are endogenously determined in equilibrium by using a consistent specification of the government budget process.\(^1\) We make two contributions to this literature. First, we model externalities and public goods in such a way that our analysis can be extended to a large set of endogenous growth models allowing for market imperfections. Second, we analyze and discuss the design of optimal fiscal and financial policies.

Given the kind of externalities and public goods specified in Romer (1986) and Barro (1990), the optimal policy could be based on static Pigouvian taxes and subsidies on factors' income. The government financial problem becomes more interesting when we restrict the set of available policy instruments, by assuming that policy makers can only tax firms' net output (or households' income). We will show that, in this case, the implementation of the optimal policy requires the government to be able to borrow and lend in financial markets. Thus, the sections devoted to normative analysis will contrast the welfare properties of static and intertemporal financial policies.

An important characteristic of intertemporally efficient policies is that the government builds a positive stock of public wealth in the short-run; in the long run, the government finances its spending by managing a portfolio of public assets. The literature has typically discussed models in which the public sector accumulates net wealth either by resorting to an initial unanticipated wealth tax or by implementing a tax policy of the kind discussed by Judd (1985) and Chamley (1986). It bears emphasizing that almost no contribution on this issue has assessed the implications of intertemporal efficient policies on pricing and private portfolios. These implications are important, because they highlight a mechanism through which the government could build a positive stock of wealth without relying on taxation. By using the approach developed in this paper, we extend the discussion to the implementation of efficient policies through market incentives.

The paper is organized as follows. We present a stochastic version of the Arrow–Romer model in Section 2: we describe the equilibrium allocation for

\(^1\) Related models are discussed by Corsetti (1992b), and Turnovsky (1993, 1995).
given policy parameters, we discuss crowding out of capital by fiscal and financial policies and, finally, we analyze the design of the optimal policy. Following a similar scheme, Section 3 applies our analysis to Barro’s simple model of spending and endogenous growth.

2. A stochastic Arrow–Romer model

In the Arrow–Romer growth model, the economy-wide capital stock exerts a positive external effect on labor productivity, that is interpreted as learning by doing by Romer (1986). Parameters’ value are such that, from a social point of view, the return to capital is sufficiently bound away from zero, so that the Inada conditions do not hold.

2.1. Preferences and technology

The economy is populated by a large number of identical dynastic families, each characterized by the following utility function:

\[
E_0 \int_0^\infty \frac{C(t)^{1-R} - 1}{1 - R} \exp(-\delta t) \, dt \quad \delta > 0, \quad R \in (0, \infty),
\]

(2.1)

where \(E_0\) is the expectation operator conditional on information at time \(t = 0\), \(C(t)\) is the instantaneous rate of consumption at time \(t\), \(R\) is the elasticity of marginal utility and \(\delta\) is the rate of time preference. Instantaneous felicity only depends on consumption, so that labor is inelastically supplied by the representative agent. For the sake of simplicity, the population is assumed to be stationary (\(L(t) = L\)).

There is a finitely large number of firms that use the same technology and compete with each other in both the factors and the goods markets. The production function of the representative firm has constant returns to scale with respect to capital and labor. Because of the external effect of capital on labor productivity, this input is measured in efficiency units, rather than physical units. Net output of the \(i\)th firm is

\[
dY_i(t) = [\xi \, dt + \psi \, d\omega(t)]K_i(t)^{1-\beta}J_i(t)^\beta,
\]

(2.2)

where \(dY_i(t)\) is the instantaneous output flow, net of depreciation (\(Y_i(t)\) is cumulative output at time \(t\)), \(K_i(t)\) and \(J_i(t)\) are capital and labor inputs, \(d\omega(t)\) is the increment to a Wiener process with zero mean and unit variance, and \(\xi\) and \(\psi\) are positive constants that denote the instantaneous drift and standard deviation of productivity shocks. Note that we define the instantaneous output flow \(\text{d}Y\) instead of \(\text{d}Y\); as we model productivity shocks as Brownian motions, we use the standard notation in stochastic calculus.
As in Romer (1986), we consider an external effect of capital on labor productivity, specified by the following definition of labor efficiency units:

\[ J_i(t) = L_i(t)K(t) \]  

(2.3)

where \( L_i(t) \) are labor physical units and \( K(t) \) is the total stock of capital outstanding in the economy. According to (2.2), for a given realization of the productivity shock, the representative firm will face decreasing returns to capital. However, it is apparent that, from a social point of view, the production function is linear in capital. Defining two new variables

\[ \eta = [NL_i]^{\beta \xi} = L^{\beta \xi}, \]  

(2.4)

\[ \sigma = [NL_i]^{\beta \psi} = L^{\beta \psi} \]  

(2.5)

and substituting, the production function of the representative firm can be written as an 'AK' function of the kind discussed by Rebelo (1991) with a stochastic linear coefficient

\[ dE(t) = [\eta dt + \sigma d\omega(t)]K_i(t). \]  

(2.6)

It follows that the instantaneous social marginal productivity of capital is identically and independently distributed with mean \( \eta \) and standard deviation \( \sigma \). A growing population would induce a trend in the value of these parameters, the presence of scale effects being a well-known feature of endogenous growth models.

2.2. The public sector

We specify a general linear aggregate tax function in the following form:

\[ dT(t) = N(t) \sum_i N_i(t) dT_i(t) = N(t) \sum_i \left[ \tau dY_i(t) + \alpha \sigma K_i(t) d\omega(t) \right], \]  

(2.7)

where \( N(t) \) is the number of firms in the economy, \( T_i(t) \) is cumulated tax revenue accruing from firm \( i \), \( \tau \) is a time-invariant tax rate on expected net output and \( \alpha \) is a time-invariant rate on output exceeding (or falling short of) its expected level. Such a function describes a variety of taxation schemes characterized by contingent provisions that the government grants to firms depending on their production performance. With \( \alpha > 0 \), the effective tax rate rises above \( \tau \) in the presence of a positive productivity shock, it falls below \( \tau \) in the presence of a negative shock. For \( \tau + \alpha = 0 \), the instantaneous tax revenue is not contingent.

\[ ^2 \text{Nonetheless, at each instant in time, the mean and the variance of the distribution of net output (in level) depends on the existing capital stock. Consequently, current output has an infinite memory of past shocks, to the extent that these have influenced the rate of capital accumulation.} \]
Budget deficits are financed by issuing consols paying an instantaneous real coupon equal to $u$. Defining $B(t)$ and $q_B(t)$ as the number of consols and their price in terms of the consumption good, the dynamics of public debt is described by the following stochastic differential equation:

$$d[q_B(t)B(t)] = G(t)dt - dT(t) + q_B(t)B(t) \left[ \frac{udt}{q_B(t)} + \frac{dq_B(t)}{q_B(t)} \right],$$

where $dq_B(t)/q_B(t)$ are capital gains on consols and $G(t)dt$ is public spending on goods and services. Given the public sector solvency constraint, the value of the outstanding stock of debt must be equal to the present discounted value of the expected flow of present and future primary surpluses (revenues minus spending net of interest payments). Note that $B(t)$ is not necessarily positive, as the government can become a net creditor in the economy.

In discussing our stochastic version of the Arrow–Romer model, we will posit

$$G(t) = 0,$$

(2.9)
i.e. that government spending can only take the form of firm subsidies, included in $dT(t)$. This simplifying assumption will be dropped in Section 3.

### 2.3. Risk and wealth allocation in a competitive equilibrium

This section is devoted to the analysis of a long-run competitive equilibrium allocation for given policy parameters. The next two sections will analyze the response of the equilibrium allocation to changes in policies, and address the issue of designing the optimal policy.

Financial assets include equity shares, consols and claims to human capital. These are assumed to be freely traded in competitive markets (this assumption is relaxed in Appendix C). Thus, private financial wealth is

$$W(t) = K(t) + q_B(t)B(t) + q_H(t)H(t),$$

(2.10)

where $H(t)$ and $q_H(t)$ are units and the real price of claims to individual labor income. The consumption good is the numéraire. It should be emphasized that the market value of physical and human capital is defined as the present discounted value of the after-tax income from these assets.

With time-invariant policy parameters, technological shocks are the only source of uncertainty in our model: the after-tax equilibrium rate of return on each financial asset will only depend on these shocks; that is, it will be generated by an Itô process of the following form:

$$r_i(t)dt + \sigma_i d\omega(t), \quad i = K, B, H,$$

(2.11)

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3 The equation is derived from the discrete-time version following Merton (1990; pp.: 124–126).
where $d\omega(t)$ is the common stochastic element driving the return of all financial assets. An important implication of (2.11) is that the returns on all assets will be perfectly correlated.

With competitive markets, the return on equity shares will be the net marginal product of capital. Thus, in the case of equity shares, the parameters of the process (2.11) can be easily derived from (2.6) and (2.7)

\begin{align*}
r_K(t) &= (1 - \tau)(1 - \beta)\eta, \quad (2.12) \\
\sigma_K(t) &= [1 - (\tau + \alpha)](1 - \beta)\sigma. \quad (2.13)
\end{align*}

All other rates of return will be endogenous in equilibrium. For instance, we need to find closed-form expressions for the parameters $r_B$ and $\sigma_B$ that describe the return on consols:

\begin{align*}
\frac{dq_B(t)}{q_B(t)} + \frac{u}{q_B(t)} &= r_B(t) dt + \sigma_B(t) d\omega(t) \quad (2.14)
\end{align*}

Having posited the functional form (2.11) describing rates of return, we now write the optimization problem faced by the representative agent. We use $n_K$, $n_B$ and $n_H$ to denote the private portfolio shares of, respectively, equity, consols and human capital, so that the portfolio constraint is $n_K + n_B + n_H = 1$. Following Merton (1969), the representative agent problem is:

\begin{align*}
\text{Max} \quad & E_0 \int_0^\infty \frac{C(t)^{1-R} - 1}{1 - R} \exp(-\delta t) dt \\
\text{subject to:} \\
& dW(t) = n_e(t)W(t)[r_K(t) dt + \sigma_K(t) d\omega(t)] \\
& \quad + n_B(t)W(t)[r_B(t) dt + \sigma_B(t) d\omega(t)] \\
& \quad + [1 - n_K(t) - n_B(t)]W(t)[r_H(t) dt + \sigma_H(t) d\omega(t)] - C(t) dt,
\end{align*}

where private sector solvency excludes negative values of households' net wealth, that is, $W(t) \geq 0$. As this problem is well-known in dynamic programming, its solution is presented in Appendix A.

In order to obtain closed-form expressions describing the equilibrium allocation, we combine the first-order conditions of the representative agent’s problem with the conjectured expressions for rates of return. In addition, we need to check that the equilibrium allocation is feasible ($K(t) \geq 0$) and that the initial stock of public debt $B(0)$ is consistent with the public sector solvency constraint – implying that consols are traded at a positive price $q_B(0) \geq 0$. Leaving the analytical details of the solution to Appendix B, we describe the equilibrium allocation in terms of Eqs. (2.16)-(2.19) below.
For reasons that will become clear later, we define a certainty equivalent tax rate $\gamma$ as

$$\gamma \equiv \eta \tau - R \sigma^2 (\tau + \alpha).$$  \hfill (2.16)

Assuming the existence of an interior solution to the consumer problem, the equilibrium portfolio shares of capital and government debt are:

$$n_k = \frac{(1 - R)[\beta(\eta - R \sigma^2) + (1 - \beta)\gamma + \delta/(1 - R) - \eta + 0.5R \sigma^2]}{\beta(\eta - R \sigma^2) + (1 - \beta)\gamma + (1 - R)[\delta/(1 - R) - \eta + 0.5R \sigma^2]},$$  \hfill (2.17)

$$n_B = \frac{R \gamma}{\beta(\eta - R \sigma^2) + (1 - \beta)\gamma + (1 - R)[\delta/(1 - R) - \eta + 0.5R \sigma^2]}.$$  \hfill (2.18)

The equilibrium rate of consumption out of private wealth is

$$\frac{C(t)}{W(t)} = \frac{n_k(1 - R)}{n_k - (1 - R)} \left[ \frac{\delta}{1 - R} - \eta + 0.5R \sigma^2 \right]$$  \hfill (2.19)

These equations show that the portfolio shares and the consumption rate out of private wealth are all time-invariant in equilibrium. This is due to two features of our model: first, the utility function implies constant relative risk aversion, so that the portfolio allocation does not change with the level of wealth; and, second, the parameters of the distribution of the shock are assumed to be time-invariant. As discussed in Appendix B, these features of the model imply that the equilibrium distribution of all rates of return is also time-invariant, as is the long-run rate of capital accumulation. By using (2.19), the stationary growth path can be written as follows:

$$\frac{dK(t)}{K(t)} = \frac{dY(t) - C(t) dt}{K(t)} = \left[ \eta - \frac{C}{W n_k} \right] dt + \sigma d\omega(t).$$  \hfill (2.20)

Note that, while the drift of the growth rate depends on policy parameters (through $C/W$ and $n_k$), its standard deviation only reflects technology and (by (2.5)) demography. There exists an exogenously given technological risk in the economy as a whole that cannot be reduced through tax policies.

2.4. Growth and crowding out

The reason why $\gamma$ is a certainty equivalent tax rate is apparent by inspection of equations (2.17)–(2.19). Fiscal reforms changing the long-run values of $\tau$ and $\alpha$ only alter the equilibrium allocation if they change $\gamma$. For a given value of $\gamma$, there is a continuum of tax rates that supports the same equilibrium allocation, with $\tau$ and $\alpha$ varying according to the following expression:

$$\frac{d\alpha}{dt} \bigg|_{\text{given } \gamma} = \frac{\eta - R \sigma^2}{R \sigma^2}.$$  \hfill (2.21)
This expression defines the terms of trade at which the expected return and risk of all assets can be modified without affecting budget constraints and portfolio allocations. In the case of equity shares, for instance, lowering $\alpha$ reduces their risk; raising $\tau$ lowers both the expected level and the variance of their return; while a tax reform will generally affect the equilibrium demand for equity shares, varying both $\alpha$ and $\tau$ according to (2.21) leaves the demand for this asset at its pre-reform level.

It is easy to show that both the market rate of return and the growth rate depend negatively on $\gamma$: fiscal policies can crowd in or crowd out capital accumulation. A positive certainty equivalent tax (i.e. a positive $\gamma$) will decrease growth, a subsidy (i.e. a negative $\gamma$) will have the opposite effect. Each value of $\gamma$ will correspond to a particular debt-to-capital ratio $n_B/n_K$. In general, higher long-run values of $\gamma$ will be associated with higher debt-to-capital ratios, but this is not necessarily the case. When the parameter $R$ is less than 1, an increase in taxes may reduce the quantity of capital in private portfolios, even though the debt-to-capital ratio is falling.

In conclusion, note that expression (2.21) defines the condition for implementing iso-growth, iso-consumption and iso-welfare tax reforms.

2.5. Optimal policy

Consider a benevolent government whose objective is to maximize the representative agent’s welfare, that is, to choose an efficient fiscal and financial policy that is feasible and consistent with the market determination of prices and quantities. Without any constraint on the set of feasible policies, the problem would be rather simple: in an Arrow–Romer model, capital exerts an external effect on labor productivity that drives a wedge between the private and social costs of capital and labor; by inspection of expressions (2.2), (2.12) and (2.13), it is apparent that a Pigouvian scheme of taxes on labor income serving to finance capital income subsidies could correct the inefficiency. The budget would be instantaneously balanced; the stock of public debt would be zero. The issue we address in this section is whether there is any policy capable of supporting the first-best allocation when we exclude factor income taxes from the set of feasible policy instruments. In what follows, we will look for an efficient policy conditional upon the tax function (2.7).

As a first step, we solve an unconstrained social planner problem; this is the problem of maximizing the expected utility of the representative agent subject to technology and endowment constraints. Since leisure is not an argument in the utility function, the social planner problem only consists in choosing a positive consumption rate that maximizes the expected utility (2.1), subject to the resource constraint (2.20), the initial capital stock constraint $K(0)$ and the feasibility

\[4 \text{The optimal tax on labor is } \beta^{-1}, \text{ while the optimal subsidy to capital income is } -(1 - \beta)^{-1}.\]
constraint \( K(t) \geq 0 \). Using the method described in Appendix A, it can be shown that the first-order condition of this problem\(^5\) is

\[
\frac{C(t)}{K(t)} = R^{-1} \{ (R - 1)[\eta - 0.5R\sigma^2] + \delta \}.
\]  

(2.22)

A long-run efficient policy can be derived by equating this expression to the equilibrium private consumption rate out of total capital, \( C/Wn_K \), that is given by the ratio of (2.19) and (2.17). In terms of the certainty equivalent tax rate \( \gamma \), the resulting optimal policy is

\[
\hat{\gamma} = \frac{-\beta}{(1 - \beta)}(\eta - R\sigma^2).
\]  

(2.23)

By evaluating \( \hat{\gamma} \) at \( \alpha = 0 \), we can see that such a policy corresponds to a flat-rate subsidy to production

\[
\hat{\gamma}|_{\alpha=0} = \frac{-\beta}{(1 - \beta)} < 0.
\]  

(2.24)

In a market economy with such a policy in place, the production subsidy completely offsets the distortions induced by the external effect of capital on labor productivity. In this respect, we need to stress that this result is conditional on the supply of labor being inelastic. With an elastic labor supply, a production subsidy would distort the static labor-leisure decision. Being short of one instrument, policy makers would only be able to target a second-best allocation.

2.6. A discussion of the optimal policy

From the point of view of wealth composition, it is easy to show that the share of capital in wealth in an economy with the optimal policy in place is \( n_K[\hat{\gamma}] = 1 \). Private agents perceive that their real net wealth coincide with the privately owned stock of capital in the economy. Human capital is still a positive component of wealth, but its value is offset by the share of public sector assets in the market portfolios: \( n_H[\hat{\gamma}] = n_P[\hat{\gamma}] \). With \( B < 0 \), the government is a net creditor vis-à-vis the private sector.

How can the government issue negative public debt? In the absence of constraints on tax rates the government can resort to an initial capital levy that endows the public sector with a positive stock of wealth. In terms of our policy, the initial capital levy is instantaneously complemented by a portfolio reshuffle, that turns public equity into public credit, i.e. liabilities towards the government that the private sector as a whole incurs when agents buy equities back from the

\[^5\]The transversality condition required for a well-defined solution is

\[
\delta > (1 - R)(\eta - 0.5R\sigma^2)
\]
public sector. However, the expropriatory nature of a capital levy conceals an important feature of the optimal policy, that is implied by the Pareto-improvement associated with it. We discuss this feature by analyzing an alternative way of building a positive stock of public wealth, consisting of a sale to firms of the right to receive subsidies over time.

Consider a market in which the government sells certificates that give firms the right to receive production subsidies at the optimal rate (2.23). Clearly, if taxes and subsidies were lump-sum, trading certificates would be neutral as regards the equilibrium allocation, and firms and households would be indifferent between buying and not buying them. However, when subsidies are distortionary, we should bear in mind that a widespread acquisition of subsidy certificates changes the equilibrium prices and increases private utility. Buying these certificates provides private agents with the opportunity to arbitrage utility across two different market allocations. Such an arbitrage can be illustrated by looking at the equilibrium prices of the optimal flow of subsidies. Since the introduction of income subsidies changes the intertemporal price of consumption, the present discounted value of the optimal flow of subsidies is lower in an economy with the optimal policy in place, relative to the market allocation without subsidies.  

The government can therefore sell subsidy certificates at a price that is below their presented discounted value in the suboptimal equilibria.

While a single firm may finance the acquisition of these certificates either by borrowing from the financial markets or by selling part of its capital endowment, in a closed economy the private sector as a whole will become a net borrower from the government. Thus, in an equilibrium with subsidy certificates, the private sector endows the government with a positive stock of wealth on a voluntary basis.

It should be emphasized that the argument supporting subsidy (or tax) certificates has been developed under the same set of assumptions as in most contributions on intertemporal taxation: agents have an infinite horizon; information is symmetric; the public sector maximizes the utility function of the representative agent and can credibly precommit to a given policy. Under these conditions, policy makers can build a net asset position making use of private incentives rather than an expropriatory tax on private wealth.

After a capital levy, or after a sale of subsidy certificates, our economy immediately jumps to a stationary growth path. When (a) there is no market for our certificates and (b) there are constraints on the initial rate of capital taxation, the policy problem could be addressed as in Charnley (1986) and Judd (1985), i.e. in terms of a restricted planner problem. Consistent with these papers, we

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6 This can be seen by evaluating the optimal stream of subsidies \( yK \) using the return on capital as the discount rate.

7 In this case, we should pose a restricted planner problem: that is, while internalizing all externalities, we should constrain the government to use only distortionary tax rates.
should posit a second-best policy problem and verify the efficiency of the follow-
ing bang-bang solution: the tax rate on income is set at the maximum possible value over a finite interval (between $t = 0$ and $t = T$), then reduced to its optimal negative value.\footnote{The intertemporal tax problem is discussed in many contributions, including Jones et al. (1993), Zhou (1993) and Judd (1995). For some model specifications, the long-run optimal tax rate may actually be positive, although small. See the two-sector model presented in Jones et al. (1993).} While we leave the analysis of such a policy in our mean-variance model to future research, we stress that, according to our conjectured optimal policy, the initial period of high tax rates coincides with a build-up of public wealth. At the time of the policy switch from a regime of high positive taxation to a regime of negative taxation, the present discounted value of public spending will be equal to the market value of government net credit. Along a stationary growth path (for $t \geq T$), the market allocation will be characterized by (2.23) and $n_H(\tau) = n_B(\tau)$. Both public credit-to-capital ratio and the stationary growth rate will be the same as in the first best allocation. Nonetheless, because of distortions induced by the initial period of high income taxation, the level of private consumption will be permanently lower.

3. A stochastic model of spending and growth

The Arrow–Romer model discussed in the previous section contains all the essential elements of our mean-variance approach to modeling stochastic AK economies. In this section, we will apply our approach to another popular AK model that has been extensively used in the analysis of public finance issues: Barro’s model of government spending and endogenous growth (Barro, 1990). The salient features of Barro’s specification are the following: the government supplies public goods which are productive; in a market economy, public spending has an external effect on labor productivity, which generates production rents and raises the wage rate above its social level.

3.1. The structure of the economy

The model can be analyzed by using the methodology presented in Section 2 and the related Appendices (see also Corsetti, 1992a). In order to stress analogies, we will use the same notation, except that variables specifically referring to the Barro model will be written with a tilde.

As in Section 2, labor is measured in efficiency-units. We replace (2.3) with the following new definition of labor efficiency units

$$\tilde{J}_t \equiv L_t(t)G(t).$$

(3.1)

Such a definition implies that, in a market economy, production rents induced by government services are appropriated by labor.
The resource constraint of the economy will now include government consumption. Dropping assumption (2.9) and defining $g(t)$ as government spending per unit of capital (that is $G(t) = g(t)K(t)$), we can write
\begin{align*}
dK(t) &= dY(t) - C(t) dt - G(t) dt \\
&= \left\{ [\xi L^\beta g(t) - g(t)] - \frac{C(t)}{K(t)} \right\} K(t) dt + \psi L^\beta g(t) K(t) d\omega(t). \\
&= \left\{ [\xi L^\beta g(t) - g(t)] - \frac{C(t)}{K(t)} \right\} K(t) dt + \psi L^\beta g(t) K(t) d\omega(t). \tag{3.2}
\end{align*}

For a given realization of the shock, government consumption contributes to net output at a decreasing rate ($L^\beta g(t)[(\xi dt + \psi d\omega(t))$); yet, it consumes resources at a constant rate ($[g(t)K(t)] dt$). Allowing for such a specification of government spending, we replace $\eta$ and $\sigma$ in (2.4) and (2.5) with two new variables $\tilde{\eta}$ and $\tilde{\sigma}$, defined as follows:
\begin{align*}
\tilde{\eta} &\equiv \xi L^\beta g(t), \\
\tilde{\sigma} &\equiv \psi L^\beta g(t). \tag{3.3, 3.4}
\end{align*}

Provided we use the newly defined variables $\tilde{J}$, $\tilde{\eta}$ and $\tilde{\sigma}$, the economy can be described in terms of the same structural equations of the Arrow–Romer model, (2.6)–(2.14).

### 3.2. Equilibrium allocation

In this section, we analyze the equilibrium allocation conditional on the assumption that the government keeps public spending constant relative to capital, $g(t) = g$. Subsequently, we will show that it is indeed optimal for the government to do so. *Mutatis mutandis* (in particular, re-defining the certainty equivalent tax rate), the expression for the share of capital is identical to (2.17):
\begin{align*}
\tilde{n}_K &= \frac{(1 - R)[\beta(\tilde{\eta} - R\tilde{\sigma}^2) + (1 - \beta)\tilde{y} + \delta/(1 - R) - \tilde{\eta} + 0.5R\tilde{\sigma}^2]}{\beta(\tilde{\eta} - R\tilde{\sigma}^2) + (1 - \beta)\tilde{y} + (1 - R)[\delta/(1 - R) - \tilde{\eta} + 0.5R\tilde{\sigma}^2]]. \tag{3.5}
\end{align*}
The portfolio share of consols in private wealth differs from (2.18) only in the numerator:
\begin{align*}
\tilde{n}_B &= \frac{R[\tilde{\eta} - g]}{\beta(\tilde{\eta} - R\tilde{\sigma}^2) + (1 - \beta)\tilde{y} + (1 - R)[\delta/(1 - R) - \tilde{\eta} + 0.5R\tilde{\sigma}^2]]. \tag{3.6}
\end{align*}
The rate of private consumption out of wealth is given by
\begin{align*}
\frac{C(t)}{W(t)} &= \frac{\tilde{n}_K(1 - R)}{\tilde{n}_K - (1 - R)} \left[ \frac{\delta}{1 - R} - \tilde{\eta} + 0.5R\tilde{\sigma}^2 \right], \tag{3.7}
\end{align*}
and the growth rate of the economy by
\begin{align*}
\frac{dK(t)}{K(t)} &= \left[ \tilde{\eta} - g - \frac{C}{W\tilde{n}_K} \right] + \tilde{\sigma} d\omega(t). \tag{3.8}
\end{align*}
Consistent with the Arrow–Romer model of Section 2, for a CRRA utility function and a time-invariant distribution of the productivity shock, the portfolio shares (3.6) and (3.5), the rate of consumption (3.7) and the distribution of the growth rate of the economy (3.8) will all be time-invariant in equilibrium. The certainty equivalent fiscal variable \( \gamma \) captures the equilibrium trade-off between different moments of the distribution of returns. Changing \( \tau \) and \( z \) for a given \( \gamma \) does not alter the relative demand for assets.

3.3. Optimal policy

Consider now the social planner’s problem. As opposed to Section 2, there are now two choice variables: one is the rate of private consumption, the other the rate of public consumption, which, from a social perspective, is an input in the production process. In a first-best allocation, government and private consumption satisfy

\[
\gamma \left( 1 - \beta (\hat{\eta} - R\hat{\sigma}^2) \right) = 0, \quad (3.9)
\]

\[
\frac{C(t)}{K(t)} \bigg|_0 = R^{-1} \{ (R - 1)(\hat{\eta} - g - 0.5R\hat{\sigma}^2) + \delta \}. \quad (3.10)
\]

According to the first expression, the optimal size of government spending is a function of the parameter of relative risk-aversion \( R \) as well as of the variability of output, \( \hat{\sigma} \). This means that policy makers will choose the size of the public sector taking into account that its activities modify the expected growth and variability of private consumption. The first-order condition (3.9) is a generalization of the result in Barro (1990), stating that the spending rate should maximize the growth rate of the economy (as is the case when \( \sigma = 0 \)). Note that, with risk-neutral households \( (R = 0) \), the optimal spending rate will maximize expected growth. As the parameters describing the output process are assumed to be constant, \( g_o \) will be time-invariant: a policy supporting a first-best allocation will be characterized by a constant rate of public consumption out of capital.

Positing \( g = g_o \), and using the same approach as in Section 2.5, we can derive the optimal policy by equating the rate of consumption in a competitive allocation (given by the ratio of (3.7) to (3.5)) with the first-best consumption rate (3.10) and solving the resulting equation for the certainty equivalent tax rate; by (3.9), we know that such a rate will be zero at an optimum:

\[
\gamma_o = \frac{g_o}{1 - \beta (\hat{\eta} - R\hat{\sigma}^2)} = 0. \quad (3.11)
\]

With such a policy in place, the optimal share of capital in private wealth is equal to \( \hat{n}_K[\gamma_o] = 1 \).

It is instructive to compare (3.11) to the optimal policy in the Arrow–Romer model (2.23). In both models, the optimal policy leads to a complete internalization of the externality; private net wealth coincides with the outstanding stock
of capital, while public net wealth is positive and equal to the value of human capital \( n_H[\gamma] = -n_H[\gamma] \). However, while in Section 2 we had an optimal provision of production subsidies, (3.11) results in the optimal tax rate \( \gamma \) being equal to zero. In Section 2, the production subsidy offsets the external effect of capital on labor productivity; in this section any certainty equivalent tax (or subsidy) would insert a wedge between the social price and the private price of capital. Note that (3.11) is consistent with non-zero values for \( \alpha \) and \( \tau \).

When government spending is an input in production, the optimal solution can be interpreted as follows: at an optimum, a positive asset position corresponds to the present discounted value of the current and the expected future flow of productive services supplied by the government to private firms. Consistent with our previous discussion, the optimal policy could be implemented by means of a capital levy, or through a sale of tax certificates, allowing firms to buy the right not to pay (distortionary) taxes over time.

4. Conclusions

This paper has developed a portfolio approach to the analysis of fiscal and financial policy in endogenous growth models. Drawing on the intertemporal CAPM in continuous time, the analysis focuses on the role of budget policies in the process of capital accumulation, stressing their general-equilibrium effects on the vector of returns, wealth composition and aggregate risk-taking.

Over the past few years, the endogenous growth literature has extensively analyzed long-run issues in the design of fiscal and financial policy. Many of the results derived in this paper can thus be related to the logical core of this literature. Nonetheless, carrying out policy analysis in terms of portfolio theory offers some specific advantages. First, the model directly maps fiscal and financial policies on the equilibrium composition of private wealth. It therefore provides important insights on the characteristics of intertemporal financial strategies involving public borrowing and lending. In this respect, our portfolio approach provides a comprehensive characterization of the welfare properties of public debt and 'credit'.

Second, by building a mean-variance model of growth, the approach provides a relative simple characterization of the risk-return profile of real and financial assets. In general equilibrium, the vector of returns can respond to fiscal and financial policy in surprising ways. In order to assess the effect of budget reforms on the opportunity set of private agents, one should identify general-equilibrium certainty-equivalent fiscal parameters. The portfolio approach considerably simplifies the problem of identifying such parameters.

In this paper, we have applied our portfolio approach to two well-known models of endogenous growth: the Arrow–Romer model with an external effect of capital on labor productivity, and the Barro model of government spending and
growth. In both specifications, we addressed the issue of designing the efficient policy by contrasting static (Pigouvian) tax schemes to intertemporal tax schemes.

A crucial feature of the intertemporal optimal policy is that in the long-run, government spending is financed by using revenue from asset management, rather than from distortionary taxation. The implementation of the policy thus requires a positive long-run stock of public wealth. Such a feature of intertemporal optimal taxation is clearly controversial, and should be assessed within the framework of general models addressing time-consistency issues or allowing for asymmetric information.

Regarding possible extensions of the analysis, the portfolio approach to modeling endogenous growth has been applied to such different issues as capital income taxation, taxation and risk-taking, and inflation and growth (Corsetti 1992a). Further extensions of the policy analysis could allow for time-varying fiscal parameters, so as to study the transition paths to the long-run balanced growth. Finally, the model could include international trade of goods and assets with the aim of exploring the role of budget policy in affecting international portfolio diversification.

Appendix A

This Appendix briefly discusses the solution of the representative agent’s problem (13). Consider the function

\[ V[W(t), t] = \max_{\{C, a\}} E_t \int_t^{\infty} \frac{C(s)^{1-R} - 1}{1-R} \exp(-\delta s) ds. \]

This function must satisfy the Bellman equation

\[ 0 = \max_{C(t), a(t)} E_t \left[ \frac{C(t)^{1-R} - 1}{1-R} \exp(-\delta t) + V_t + V_W dW + \frac{1}{2} V_{WW} dW^2 \right] \]

and the transversality condition \( \lim_{t \to \infty} E_t V[W(s), s] = 0 \). Positing \( V[W(t), t] = \exp(\delta t)\chi^{-R}W(t)^{1-R} / (1-R) \) and assuming that an interior solution exists, the optimality conditions of our problem are

\[ C(t) = \chi(t) W(t), \quad \chi \geq 0, \]

\[ [r_K(t) - r_H(t)] - R[o_s(t) - \sigma_H(t)][n_K(t)\sigma_s(t) + n_B(t)\sigma_B(t)] + (1 - n_B(t) - n_K(t))\sigma_H(t)] = 0, \]

\[ [r_B(t) - r_H(t)] + R[\sigma_B(t) - \sigma_H(t)][n_K(t)\sigma_s(t) + n_B(t)\sigma_B(t)] + (1 - n_B(t) - n_K(t))\sigma_H(t)] = 0. \]
The first condition implies that the representative agent consumes out of wealth at the instantaneously deterministic rate $\chi(t)\,dt$. This rate must be non-negative, otherwise expected utility would not be bounded and – as shown in Merton (1969) – the transversality condition would be violated. The following two expressions are standard first-order conditions of the portfolio problem in an intertemporal CAPM when all returns are perfectly correlated.

**Appendix B**

This appendix discusses the solution of the Arrow–Romer model in Section 2. We focus on a market allocation conditional on given time-invariant policy parameters $\tau$ and $\alpha$. Starting from the first-order conditions of the consumer problem (Appendix A), a closed-form solution requires knowledge of closed-form solutions for the rates of return on financial assets. While the return on equity shares can be easily determined by using the first-order condition of the firm problem – that is, using the marginal product of capital – the rates of return on consols and human capital are endogenous in equilibrium.

Since the distribution of the technological shock per unit of capital is stationary, while preferences exhibit constant relative risk aversion (CRRA), we conjecture the existence of an equilibrium where the distribution of all rates of return is time-invariant, while the portfolio shares are independent of the level of wealth and time-invariant. Differentiating the portfolio shares according to our conjecture and dividing by private wealth, we obtain

$$\frac{dW(t)}{W(t)} = \frac{d[q_B(t)B(t)]}{q_B(t)B(t)} = \frac{d[q_H(t)H(t)]}{q_H(t)H(t)} = \frac{dK(t)}{K(t)}.$$ 

In the equilibrium we are focusing on, all components of wealth grow at the same rate, which is equal to the rate of capital accumulation. As all portfolio shares are kept constant, we can use the above expression to obtain

\[
\begin{align*}
r_B(t) &= \left\{ \eta - \frac{C}{K} \right\} + \tau \eta(n_K/n_B), \\
\sigma_B(t) &= \{\sigma\} + \sigma(\tau + \alpha)(n_K/n_B), \\
r_H(t) &= \left\{ \eta - \frac{C}{K} \right\} + \frac{(1 - \tau)\beta n_K}{1 - n_K - n_B}, \\
\sigma_H(t) &= \{\sigma\} + \sigma(1 - \tau - \alpha)\frac{\beta n_K}{1 - n_K - n_B}.
\end{align*}
\]
In the above expressions, the rates of return on consols and human capital can be seen as the sum of two components: the first is the growth rate of the economy, the second a measure of the instantaneous income rate paid out by each asset, measured in terms of consumption goods. The first component is demand-determined and reflects capital gains and losses when agents try to adjust their portfolio shares in response to productivity shocks affecting the rate of growth of the economy.

Note that the equilibrium return on consols is independent of the coupon \( u \). This is because, along a stationary growth path, fiscal policy determines the flow of primary surpluses. For a given path of primary surpluses, by the public sector solvency constraint, an increase in the coupon \( u \) simply raises the rate of debt issuance, bringing about capital losses on outstanding consols that offset the increase in the coupon. In the analysis, we assume that the coupon \( u \) is consistent with the long-term equilibrium interest rate.

The solution in the text is derived by using the optimality conditions of both the consumer and the firm problems, the expressions for \( r_K, \sigma_y, r_B, \sigma_B, r_H, \) and \( \sigma_H \) shown above, and the Bellman equation. The resulting system of equations describes the equilibrium allocation for a given set of fiscal parameters. It is easy to verify that, consistently with our conjecture, the drift and standard deviation of the rates of return on consols and human capital are indeed constant in equilibrium.

**Appendix C**

Would our results in Section 2 change if we ruled out a market for human capital? In this respect, it is important to stress a feature of representative-agent models. Incomplete markets do not reduce the opportunities for risk diversification. By construction, there are no differences in the evaluation of state-prices, so that markets are *effectively complete* (Ingersoll, 1987). Thus, ruling out a market for human capital has no major implication for our results.

Suppose that such a market is indeed missing and that, for the sake of simplicity, wages are paid out at a deterministic rate equal to the expected marginal product

\[
  w(t) \, dt = (1 - \tau) \beta[\xi \, dt] \left[ \frac{K_t}{J_t} \right]^{1-\beta}.
\]

Our analysis can be carried out in identical terms by simply redefining the set of choice variables in the consumer problem: first, the relevant rate of consumption is no longer \( C(t) \) but the difference between the consumption rate and labor income, net of taxation (as in Merton, 1990, pp. 143–144); second, private portfolios will include only two financial assets, equity shares and consols.
References


Corsetti, G., 1992b, A portfolio approach to endogenous growth: Eaton's model revisited, Economic Growth Center at Yale University, Center Discussion paper no. 678.


