Endogenous Shocks and Evolutionary Strategy: Application to a Three-Players Game

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Abstract

An evolutionary game with three players – trade unions, financial investors and firms – is presented where each player has a short-term and a long-term maximizing strategy at hand. The short-term strategy maximizes current payoffs without taking into account benefits from future cooperation while long-term strategies depend on the cooperative behavior of the other players. We first determine equilibria arising in the static game and determine under which conditions long-term cooperation may emerge. We then endogenize the stochastic environment, making it subject to the strategies selected and show how additional equilibria and strategy cycles arise in an evolutionary set-up.

1 Introduction

Evolutionary game theory has gained an important place in economic modeling through the last two decades. The possibility of analyzing strategic interactions in

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a low-rationality framework and the elimination of non-strict Nash equilibria has increasingly attracted economists to follow this approach. New developments in stochastic evolutionary game theory (SEGT) – first suggested by Foster and Young [5] – have moreover allowed to narrow even further the possible outcomes – at least in the very long-run.

Especially when analyzing the emergence and change of institutional arrangements in societies, this last approach has now gained some authority: social customs, patterns of interaction and adoption of standards have been analyzed successfully by SEGT. Researchers in the social sciences often face situations where the adoption of one particular strategy by one player increases the marginal returns of this – or some related – strategy for other players: a case for strategic complementarities. In this situation, coordination problems easily arise and SEGT allows the singling out of the equilibrium that is characterized by risk dominance, a concept that makes sense when allowing for very long-run developments of strategic configurations.

However, as has been put forward by Bergin and Lipman [2], the results these models generate do not in general survive small modifications of the underlying stochastic processes. In particular, state-dependent stochastic processes will not necessarily allow selection of the risk-dominant equilibrium in the long-run.

Starting from this insight, the following paper proposes an application of a three-player coordination problem to analyze the strategic interactions between trade unions, banks and firms. In particular, by making the stochastic process by which the game's payoffs are selected completely endogenous to the strategic configurations we can go even beyond the initial result of Bergin and Lipman's paper, showing that even the underlying Nash equilibria no longer need to exist: consequently, no (stochastically) evolutionary stable strategies will arise.

In the model presented here, a coordination problem between trade unions, financial intermediaries and firms arises due to a multi-dimensional strategy space. Firms have access to a long- and a short-term technology where the productivity of the long-term technology crucially depends on a stable labor input: as soon as the firm is obliged to lay-off parts of its workforce, the technology does not yield any profits any more. The short-term technology is more flexible and allows an easy rotation of the workforce which can be used as a strategic weapon against wage demands.

Workers, on the other hand, have the possibility of joining a trade union and bargaining over wages (only) or making use of an additional (costly) instrument to increase their employment stability: in our setting we suggest that this second element is an investment in their human capital that helps to reduce the idiosyncratic risk at the plant level.

Lastly, financial investors have the choice to invest directly through the stock market or to join (or form) a bank. The first choice gives them full access to the dividend flow whereas in the second case they have to bear intermediation costs.

Both strategies have different impacts on the risk distribution and the availability of information in the economy.

The marginal return to each type of strategy (either long or short term) is increased when at least one of the other players adopts the same strategy type (and even more so when both remaining players do). Hence, a coordination problem arises with equilibria that can be Pareto-ranked.

This three-players game may be solvable by using the theory of global games[1]; in these games, the risk dominant equilibrium is selected. However, the analysis of the macroeconomy suggests that interdependencies between microeconomic actors and production pools may have considerable impact on the risk structure of the economy and hence on the strategic choices that institutional actors (and firms) undertake. More importantly, the endogenous nature of the stochastic process may even affect the Nash characteristic of some of these equilibria and hence will no longer allow the application of global games or SEGT theorems.

Introducing institutional questions like these into economic modeling has gained importance in economics in recent years, especially in the literature on economic growth. Authors have become aware that the underlying mechanisms (externalities, complementarities) of these models may be influenced by factors which had not been included in standard economics until then. Moreover, historical accounts like Altvater, Hoffmann and Semmler [1] or others, suggest that it is institutional change as much as the mere existence of institutions that may influence the long-run evolution of a country's economic performance.

The paper is organized as follows: In the next two parts we analyze endogenous labor market relations and the corresponding technology choices and explore the resulting strategic game between firms, trade unions and financial investors. In section four we open the analysis for macroeconomic relations and show how and under what conditions institutional cycles can arise. A final section concludes the discussion.

2 The Stochastic Environment

The economy is composed of N production poles each containing a given number n of employees. The employees may be able to switch between these poles while they are "off" in order to find the optimal (i.e. corresponding the best to their preferences) pole with respect to the wage contract proposed at that pole. At each point in time, one of these poles is "on", i.e. organizes a game between workers, managers and its financiers in order to figure out the wage and debt contract which will be utilized in the following period where the pole is "off" (in the sense of the game play, not in the sense of production).

Once the contract has been determined, the pole starts production; if no contract agreement could be found, the pole does not produce anything until the next time

[1] There exists an important resemblance between global games and SEGT ([3,8]).

it plays the game. Moreover, we assume time consistency, i.e. that the players are committed to the strategies they decided on before the production process.

Production underlies idiosyncratic shocks u_{it} with $u_{it} \sim G_i(u_{it}|h_i, T_i)$ supposed to be stationnary and $(\partial G_i/\partial h_i) < 0$, $(\partial^2 G_i/\partial h_i \partial T_i) < 0$ where h_i denotes the workers' human capital and T_i the firm's technology choice to be determined later; human capital improves the average realization of the shock but less so for less specific technologies (with lower T_i). Moreover, idiosyncratic shocks are uncorrelated among pools, i.e. $cov(u_i, u_j) = 0 \; \forall \; i \neq j$.

The firm faces a demand function depending on aggregate income, i.e. aggregate wages and profits in each period. If the pools are not interconnected then the overall uncertainty simply equals the productivity uncertainty u_{it}. However, if pools are interconnected, then a single pool faces a collective risk through aggregate demand. Given that pools are symmetric this can be written as $\eta_{it} = (1/N)\sum_{j \neq i} \Pr(\varepsilon_{jt} \geq \underline{\varepsilon}_j) \cdot \varepsilon_{jt}$, where $\varepsilon_{it} = (1/N)u_{it} + \eta_{it}$ stands for the overall risk a firm faces and $\Pr(\varepsilon_{it} \geq \underline{\varepsilon}_j)$ for the probability that pool j reaches a certain minimum state $\underline{\varepsilon}_j$ (which will be determined later on). Cumulative distribution functions are given by $F_i(\varepsilon_{it})$ and $H_i(\eta_{it})$, both depending on the distribution functions of all idiosyncratic shocks.

After contracts have been determined, production starts yielding expected outputs y_i^e until the shock reaches some minimum state $\underline{\varepsilon}_i$ below which financial investors are no longer willing to keep their engagement at pool i. In this case, the firm will be liquidated. The exit probability can be determined as:

$$s_i = 1 - \Pr(\varepsilon_{it} \geq \underline{\varepsilon}_i) = \int_0^{\underline{\varepsilon}_i} dF_i(\varepsilon_{it}). \quad (1)$$

3 Firms, Banks and Workers: Technological Choice

In a first step, we concentrate on the microeconomic relations only, leaving out pool interdependency and focusing only on idiosyncratic shocks. We will come back to the aggregate shock when we look at the macroeconomic links. In the following we therefore drop pool indices i.

3.1 Technology and profits

Technology

The firm has to make a technology choice $T \in [0, 1]$ with technologies ranging from completely unspecific ($T = 0$) to completely specific ($T = 1$). The more specific a technology is, the lower will be its resale value in case of liquidation V^L, $V^L = V(T)$, $V' < 0$. To be precise, we want to assume that $V^L(0) = V^{\max} > 0$ and $V^L(1) = 0$. The installed technology yields the expected output $y^e \equiv Ey = p(h)y(T)$ where $y' > 0$ and $p = \int_0^\infty u \cdot dG(u|\, h, T)$ and $p'(h) > 0$, i.e. the impact of human capital, h, on p will be more important the more specific the technology is.

The lower the degree of specificity, the less important will be specific human capital investment to use the technology; therefore the workforce can easily be replaced. In case of wage negotiations firms could then use outside labor to replace parts of the insider force, increasing therefore its bargaining position. This will reduce the probability for workers to find a job covered by a collective agreement by $q = q(T, \xi)$ with $q_T < 0$, $q_\xi > 0$, $q(0, \xi) = \overline{q}(\xi) \in (0, 1)$, $q(1, \xi) = 0$, i.e. $q(T, \xi)$ measures the reduction in workers' bargaining power due to the availability of outside labor. Exogenous factors (e.g. legislation, immigration) may restrict the maximum amount of this outside labor pool that can be employed – even in the case of a completely un-specific technology – to a value $\overline{q}(\xi)$ less than one. In particular, we consider ξ as being *net* immigration [2] and analyze in the following how the equilibrium behavior changes with this parameter.

Profits

In each period the firm expects to earn a return depending on its technology choice, $y^e = p(h)y(T)$, spending w to hire workers and facing a probability $s(\kappa)$ to be liquidated due to weak performance and impatient financial investors. Denoting r the interest rate, the firm's Bellman equation can be set up as:

$$r\pi = p(h)y(T) - w + s(\kappa)[V^L(T) - \pi] + \dot{\pi}$$

which can be rewritten at the steady state (where $\dot{\pi} = 0$) as:

$$\pi = \frac{p(h)y(T) + s(\kappa)V^L(T) - w}{r + s(\kappa)}. \qquad (2)$$

As can be easily seen from this formula, all the cross derivatives between the strategic variables are positive leading to strategic complementarities:

$$\frac{\partial^2\pi}{\partial h \partial T} > 0; \; \frac{\partial^2\pi}{\partial h \partial \kappa} > 0; \; \frac{\partial^2\pi}{\partial T \partial \kappa} > 0. \qquad (3)$$

3.2 Trade unions and wage bargaining

In order to determine the wage bargaining position for workers we have to consider their maximization problem. Workers will obtain W when being employed and covered by a collective wage agreement. With probability $s(\kappa)$ the firm is liquidated

[2] Net immigration is influenced by a variety of factors including costs of immigration, perceived benefits abroad and at home, etc. which will not be taken up here in detail.

and they lose the job, obtaining their unemployment value U benefiting from some outside opportunity R. Workers can spend η to invest in their human capital h; this investment will be lost when unemployed but can be recovered on a job. Being unemployed, they will find a new unionized job with probability $\theta - q(T, \xi)$. Therefore, the two states in which the worker can be, have the following values:

$$rW = w + s(\kappa)(U - W) + \dot{W},$$
$$rU = R - \eta h + (\theta - q(T, \xi))(W - U) + \dot{U}.$$

Again placing ourselves at the steady state, the rent to be earned from being employed can be written as:

$$r(W - U) = w + \eta h - R + (s(\kappa) + \theta - q(T, \xi))(U - W)$$
$$\Leftrightarrow W - U = \frac{w + \eta h - R}{r + s(\kappa) + \theta - q(T, \xi)}. \tag{4}$$

We are assuming a Nash bargaining game with the firm's fallback position (i.e. in the case no agreement can be found) to be normalized to zero and union's bargaining power σ. Both bargaining parties therefore have to select a wage w^b such as to maximize:

$$w^b = \arg\max \pi^\sigma (W - U)^{1-\sigma}.$$

The optimal solution to this problem has to satisfy the following first-order condition:

$$(1 - \sigma)\pi = \sigma(W - U)$$

which can be rewritten as – using (2) and (4):

$$(1 - \sigma)\frac{p(h)y(T) + s(\kappa)V^L(T) - w^b}{r + s(\kappa)} = \sigma \frac{w^b + \eta h - R}{r + s(\kappa) + \theta - q(T, \xi)}$$

which can be used to calculate the bargained wage as:

$$w^b = \frac{(1 - \sigma)(p(h)y(T) + s(\kappa)V^L(T))(r + s(\kappa) + \theta - q(T, \xi)) + \sigma(R - \eta h)(r + s(\kappa))}{r + s(\kappa) + (1 - \sigma)(\theta - q(T, \xi))}$$

With the impact of human capital on the distribution of the outcome of the production process being low for short-run technologies, investment in human capital will depress the bargained wage for short-term horizon production processes. Moreover, whenever firms can use outside labor, $q(T, \xi)$, it will have a dampening effect on wage demands.

3.3 The financial relation

The financial relation is characterized by the dispersion of ownership $\kappa \in [0, 1]$ (the higher the κ the more concentrated is the ownership). If the firm is in financial distress then the building of a coalition to refinance it will be the more difficult the more the financial contractors are dispersed, i.e. the lower is κ (see [7], pp 295–299). The more finance is concentrated in the hands of a couple of banks the higher the chance for the firm to survive even in case of low realization.

Banks are assumed to be ready to restructure the enterprise and to refinance even projects with (temporarily) low expected outcomes in order to obtain credit repayments. This ability makes them accept a profit level (temporarily) lower than that required by the stock market. In this way they secure the investment even in the presence of an adverse demand or production shock. Investors on the stock market, however, are not easily able to form a stable coalition to refinance a failing project or they may not dispose of sufficient liquidity to do it ([4]); therefore they decide to liquidate the firm.

Specifically we want to suppose that survival is guaranteed if there is only bank finance involved, $\kappa = 1 \Rightarrow s(\kappa) = 0$ (i.e. $\underline{\varepsilon} = 0$). For completely dispersed ownership, survival becomes more difficult, i.e. $\kappa = 0 \Rightarrow s(\kappa) \gg 0$ (i.e. $\underline{\varepsilon} \gg 0$). According to the technology and liquidity value assumptions, the following holds given the union's strategy:

$$\pi^e(T = 1, \kappa = 1) > \pi^e(T = 0, \kappa = 1) \text{ and}$$
$$\pi^e(T = 0, \kappa = 0) > \pi^e(T = 1, \kappa = 0).$$

Furthermore we want to make the assumption that the use of the short-term technology is efficient, i.e. $\pi^e > V^L(0)$ which can be rewritten as $\pi^e(T = 0, \kappa = 1) > \pi^e(T = 0, \kappa = 0)$.

In order to determine the resulting financial relationship that emerges from the game, we have to consider the return accruing to financial investors as being a function of the financial relation they choose. The returns for the financial investor depending on his financial relation choice and on the technology i used by the firm are supposed to be:

$$\Psi(T, \kappa = 1) = r \cdot \pi^e(T, \kappa = 1) - c^B$$
$$\Psi(T, \kappa = 0) = r \cdot \pi^e(T, \kappa = 0)$$

where c^B stands for the organization costs when forming or joining a bank and is – for the moment – considered to be exogenously given as the other pools will not interfere with the pay-offs of the playing pool.

In the following figure, the different stages of the game and the moments of bargaining have been summarized:

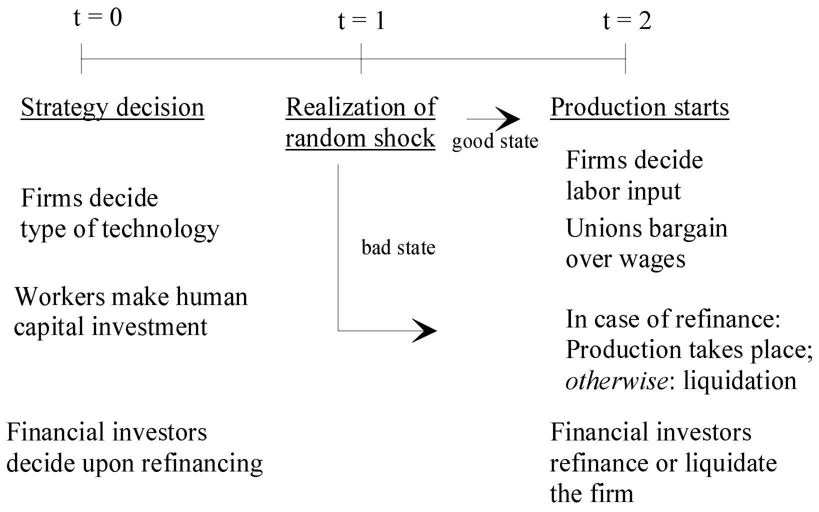


Figure 1: Timing of the game

3.4 The nash game

Having defined the various pay-offs we can now determine the equilibria of this three-players game. Firms have the choice over technologies, T, while workers choose the amount of human capital, h, they are ready to secure in the labor relation which in turn determines the time horizon of trade unions. Financial investors decide upon the degree, κ, to which they are ready to reschedule debt and to save failing firms from bankruptcy. Forming a bank comes at a cost as the restructuring of failing investment projects will necessitate time and money to proceed. Moreover, the information produced by the firm through the stock market is no longer available; the bank has to go through costly monitoring in order to obtain the necessary information.

In order to simplify the representation, we are only considering binary choices such that $T \in \{0, 1\}$, $h \in \{0, 1\}$ and $\kappa \in \{0, 1\}$ with the eight strategy combination denominated as (**S**, *s*, *FS*), (**S**, *s*, *FL*), (**L**, *s*, *FS*), (**L**, *s*, *FL*), (**S**, *l*, *FS*), (**S**, *l*, *FL*), (**L**, *l*, *FS*), (**L**, *l*, *FL*). In the following game, firms choose rows, trade unions choose columns while financial investors choose matrices.

Given the structure of the game, the following proposition can be easily verified:

Proposition 3.1. *Given the above hypotheses concerning the technology choices and profit functions and*

(i) banking costs lie in the interval:

$$p(h)y(1) - w^b > \frac{r(r+\bar{s})c^B}{\delta\bar{s}} > p(h)y(0) - w^b - rV^L(0) \ \ \forall w^b, h \quad (5)$$

with $\bar{s} = s(0)$,

(ii) *and human capital investment has the following characteristics:*

$$\frac{\eta(s(\kappa)+\sigma r)}{(1-\sigma)(r+\theta-\overline{q}(\xi))} > y(0)(p(1)-p(0)) \quad \textit{and}$$

$$\frac{\eta\sigma r}{(r+\theta)(1-\sigma)} < y(1)(p(1)-p(0)) \tag{6}$$

there exist two Nash equilibria in the above game: (***S***, *s*, *FS*) *and* (***L***, *l*, *FL*). *The game is therefore a coordination game.*

Proof. See appendix. □

Remark 3.1. Given that $\overline{q}(\xi) \in (0, 1)$

$$\frac{s(\kappa)+\sigma r}{r+\theta-\overline{q}(\xi)} > \frac{\sigma r}{r+\theta}$$

this second condition is not trivial, the coordination game structure therefore only emerges for human capital to have a sensibly different impact on the productivity of the two technologies. Moreover, notice that the first part of (6) depends on the availability of outside labor (which in turn may be influenced by net immigration). Therefore, when $\exists\overline{\xi}$ such that

$$\frac{\eta(s(\kappa)+\sigma r)}{(1-\sigma)(r+\theta-\overline{q}(\xi))} < y(0)(p(1)-p(0))\forall\xi < \overline{\xi}$$

then only the long-term strategy will emerge.

Hence, the strategic complementarities that exist between the three decision variables T, κ and h (see (3)) create a coordination problem. In order to determine the equilibrium emerging in the long-run when the players face a problem of strategic uncertainty, the theory of global games can be used to find conditions under which either the long-run or the short-run equilibrium will be chosen. In general, low banking costs, high profitability of the long-run technology and low education costs will be beneficial for the (**L**, *l*, *FL*)-equilibrium to emerge. However, in this contribution we want to concentrate on the possibility of endogenous stochastic processes as they emerge out of a macroeconomic demand spillover.

4 Demand Shocks and Fluctuations

In SEGT the stochastic environment under which behavior selection takes place is usually considered to be exogenously given although it has been shown that the results obtained that way are crucially dependent on the underlying assumptions about the stochastic processes ([2]). This is justified by the fact that boundedly

Table 1: Strategic game between trade unions, firms and financial investors.

Financial Investors	*Short term (FS)*		*Long term (FL)*	
Firms, Trade Unions	Short term (s)	Long term (l)	Short term (s)	Long term (l)
Short term (S)	$\delta\pi^e(T=0,\kappa=0)$ $W_1(T=0,h=0)$ $\pi^e(T=0,\kappa=0)$	$\delta\pi^e(T=0,\kappa=0)$ $W_2(T=0,h=1)$ $\pi^e(T=0,\kappa=0)$	$\delta\pi^e(T=0,\kappa=1)-c^B$ $W_3(T=0,h=0)$ $\pi^e(T=0,\kappa=1)$	$\delta\pi^e(T=0,\kappa=1)-c^B$ $W_4(T=0,h=1)$ $\pi^e(T=0,\kappa=1)$
Long term (L)	$\delta\pi^e(T=1,\kappa=0)$ $W_1(T=1,h=0)$ $\pi^e(T=1,\kappa=0)$	$\delta\pi^e(T=1,\kappa=0)$ $W_2(T=1,h=1)$ $\pi^e(T=1,\kappa=0)$	$\delta\pi^e(T=1,\kappa=1)-c^B$ $W_3(T=1,h=0)$ $\pi^e(T=1,\kappa=1)$	$\delta\pi^e(T=1,\kappa=1)-c^B$ $W_4(T=1,h=1)$ $\pi^e(T=1,\kappa=1)$

rational agents select strategies randomly and hence the payoff matrix remains stable, independently of the state of the game. However, as soon as the pay-off matrix integrates random elements itself, this hypothesis cannot be kept any more – as can be easily seen from the stochastic structure of a global game.

In the set-up of the model thus far, only one production pool is supposed to be 'on' at each round, determining strategies and hence supply and demand on the micro level. In a world determined by division of labor, however, demand cannot be locally satisfied but has to flow between different pools of the play grid. Letting all pools play at all times but only one pool updating its contractual environment would create exactly this kind of interdependency. In this situation, however, the shock no longer depends only on the idiosyncratic element at the firm level but also depends on the realization of all the other shocks in the economy determining aggregate demands.

In order to introduce this new element we will use the stochastic variable as defined by (1). This variable is partly determined by the survival of all of the other production pools:

$$\varepsilon_{it} = \frac{1}{N} u_{it} + \eta_{it} = \frac{1}{N} u_{it} + \frac{1}{N} \sum_{j \neq i} \Pr(\varepsilon_{jt} \geq \underline{\varepsilon}_j) \cdot \varepsilon_{jt} \tag{7}$$

which is additive separable in shocks of pool i and shocks of all the other pools and shows cumulative density function $F_i(\varepsilon_{it})$. This will have an immediate impact on the realization of the expected output:

$$p_i(h_i) = \int_0^{\infty} \varepsilon_i \cdot dF(\varepsilon_i | h_i, T_i).$$

Here h_i and T_i only have an indirect effect on $F_i(\varepsilon_i)$ through their effect on the distribution of G. Their effect on the expected ouput is therefore dampened by the pool interdependency.

In a similar way, the liquidation probability for the *n*th pool is as before:

$$s_i = \int_0^{\underline{\varepsilon}_i} dF_i(\varepsilon_{it}).$$

Notice that again as in the preceding (idiosyncratic) case the minimum state of survival $\underline{\varepsilon}_i$ is a function of κ_i such that $s_i = s_i(\kappa_i)$, $s_i' < 0$. However, while $\underline{\varepsilon}_i$ is a function of κ_i alone, due to the interdependencies the distribution function F_i is also a function of the financial relations of all the other pools:

$$\frac{\partial F_i}{\partial \kappa_i} > 0 \ \ \forall i \in \{1, .., N\}.$$

Therefore the survival probability depends also on the strategy equilibrium of the other pools:

$$s_i = s_i(\kappa_1, \ldots, \kappa_N), \ \frac{\partial s_i}{\partial \kappa_j} < 0 \ \ \forall j \in \{1, 2, .., N\}.$$

Moreover, from (7) it follows that increasing the number of pools playing $\kappa = 1$ also increases the survival probability of each individual pool; taking expectations we obtain:

$$E(\varepsilon_{it}) = \frac{1}{N} E(u_{it}) + \frac{1}{N} \sum_{j \neq i} \Pr(\varepsilon_{jt} \geq \underline{\varepsilon}_j) \cdot E(\varepsilon_{jt}).$$

Knowing that $\Pr(\varepsilon_{jt} \geq \underline{\varepsilon}_j | \kappa_j = 1, \kappa_{\neg j}) > \Pr(\varepsilon_{jt} \geq \underline{\varepsilon}_j | \kappa_j = 0, \kappa_{\neg j})$ and given a sufficiently high number of pools, N, the survival probability can be approximated by a continuous relationship.

$$k = |\boldsymbol{\kappa}| : s_i = s_i(k) \text{ with } \frac{\partial s_i}{\partial k} < 0 \text{ as } \frac{\partial E(\varepsilon_{it})}{\partial k} > 0 \ \forall k \in [0, 1]. \tag{8}$$

This is not the only way by which the characteristics of the macroeconomy influence the strategic decisions on the microeconomic level. Given the equilibrium strategies of all pools an information set $\Phi(\boldsymbol{\kappa})$ is accessible which contains the (stock market) prices of those firms that are public; this set is important in order to make interferences on the idiosyncratic shock. As long as ε_{it} contains private information which is only revealed during the play one important condition for informational efficiency is publicly (costless) observable prices. In a stationary world (which ours is) past (stock market) prices will then allow making inferences as to whether or not the firm is likely to suffer from liquidation in the next play. However, as banks learn about the private information through costly monitoring and restructuring of the firm, they are not likely to make this information public. Therefore it is plausible to assume that:

$$\Phi(\boldsymbol{\kappa}_1) \subset \Phi(\boldsymbol{\kappa}_2) \ \forall |\boldsymbol{\kappa}_1| < |\boldsymbol{\kappa}_2|.$$

However, due to the symmetry of firms, the smaller the information set the less interference one can draw about one particular firm and therefore the higher has to be the engagement of banks in monitoring their particular corporate client. The banking costs c^B can therefore no longer be thought of as exogenously given. Instead, they are likely to go up the fewer public firms there are:

$$c_i^B = c_i^B(\kappa_1, \ldots, \kappa_N), \ \frac{\partial c_i^B}{\partial \kappa_j} < 0 \ \forall j \in \{1, 2, .., N\}.$$

These results concerning linkages between pools can now be used to draw conclusions about new equilibrium behavior. In the following, two cases will be distinguished.

4.1 Opposition between unions and stock markets

Even without knowing the exact distribution of the summary variable ε_{it} we can draw conclusions on equilibria obtained in proposition (3.1). Here, we consider that the second condition of that proposition holds and hence that a coordination problem may exist. We therefore do not consider a change of the equilibrium structure due to different configurations of human capital distributions throughout the economy.

It is obvious that under the new specification of pool interdependency, satisfaction of the first condition of proposition (3.1) also depends on the equilibria obtained in the other pools; the stochastic term is not longer state-independent and the Nash character of the equilibrium may break down any time the equilibrium distribution over the whole range of production pools changes.

Proposition 4.1. *Suppose that condition (6) holds. Then:*

1. *Suppose that* $\delta\bar{s}(\boldsymbol{\kappa}')(p(1)y(1) - w^b) < r(r + \bar{s}(\boldsymbol{\kappa}'))c^B(\boldsymbol{\kappa}')$ *with* $|\boldsymbol{\kappa}'| \leq |\mathbf{1}|$ *and* $\kappa_i = 1$ *for production pool* i*. Then the long-term equilibrium (***L***, l, FL) is no longer Nash for all* $\boldsymbol{\kappa} \geq \boldsymbol{\kappa}'$*.*
2. *Suppose that* $(r + \bar{s}(\boldsymbol{\kappa}''))rc^B(\boldsymbol{\kappa}'') > \delta\bar{s}(\boldsymbol{\kappa}'')(p(h)y(0) - w^b - rV^L(0))$ *with* $|\boldsymbol{\kappa}''| \leq |\mathbf{1}|$ *and* $\kappa_i = 0$ *for production pool* i*. Then the short-term equilibrium (***S***, s, FS) is no longer Nash for all* $\boldsymbol{\kappa} \leq \boldsymbol{\kappa}''$*.*
3. *Suppose that* $\boldsymbol{\kappa}'' > \boldsymbol{\kappa}'$ *and that* $|\boldsymbol{\kappa}'| < W_L/(W_S + W_L) < |\boldsymbol{\kappa}''|$ *where* W_S*,* W_L *are measures for the workers' pay-off in the short-run and long-run equilibrium respectively. Then an interior equilibrium exists which is a saddle point.*

Proof. See appendix. □

The interior equilibrium that is hence obtained in proposition (4.1) is unstable and gives rise to a new situation where, depending on the initial conditions, the economy will end up in a situation where one of both players chooses the long-run strategy, while the other opts for the short-run strategy, following the fact that both (**L**, *l*, *FL*) and (**S**, *s*, *FS*) are repelling. Simulation 2 represents a number of trajectories that lead to either of the opposition equilibria.

More important, though, than the question of whether this interior equilibrium exists is to ask what gives rise to its existence. As we have argued in the beginning of this section, the important fact of pool interdependency and aggregate uncertainty has rarely been treated in the literature on evolutionary games. This is largely due to the fact that the source of the uncertainty has been attached to the individual non-rationality. However, in games with random payoff matrices it does not make sense to treat uncertainty as idiosyncratic and exogenous to the individual decision process. Banking activity will provide (intertemporal) insurance for the production process, and in that way set incentives for trade unions and firms to opt for the long-term strategy. However, the banking process is costly in terms of information provision and refinancing. Given a sufficiently smooth economic development, workers will have an incentive to free-ride on the insurance provided by the rest of the pools in the economy and opt for short-run gains by reducing their human capital investment and bargaining for higher wages.

Conversely it is true that an economy largely dominated by stock market finance will have more efficient information processing. Private information will become public through stock market prices giving the right investment information for financial investors. Consequently, when a sufficient number of workers guarantee through their human capital investment a smooth functioning of the economy, the individual investor will weigh banking costs of reduced information processing capacities higher than increased stabilizing of the economy. The investor will therefore opt for a more arm's length financial relation with the firm, guaranteeing a higher pay-off from the investment.

4.2 Union-led institutional cycle

Besides the effect of pool interdependency on the best reply strategy set of financial investors, workers may face a similar spillover. In the following, we want to analyze the consequences when the interdependency effects affect best reply sets of the other player population, i.e. when composition of the equilibrium set of financial investors affect best reply strategies of workers.

Going back to condition (6) one sees that the crucial factor that this condition holds is that the long-run strategy better insures the worker facing a long-run strategy by firms. However, in this new set-up an argument similar to the one before prevails: the union that decides to switch to a short-term strategy can free-ride on the stabilized economy due to high banking activity. On the other hand, in an equilibrium situation where (almost) all pools are coordinated on short-run strategies, unions may be willing to not use their bargaining power and to get at least parts of the benefits of an insurance strategy. Defining $\Theta \equiv [(1-\sigma)(r+\theta)/\eta](p(1)-p(0))$ and $\gamma \equiv \sigma(r-\overline{q}(\xi))+\overline{q}(\xi)$ we can prove the following proposition.

Proposition 4.2. *Suppose that there exist* $\boldsymbol{\kappa}', \boldsymbol{\kappa}'' \in [\mathbf{0}, \mathbf{1}]$ *such that*

$$\begin{aligned} W(T=1,\boldsymbol{\kappa},h=0) &> W(T=1,\boldsymbol{\kappa},h=1) \\ &\Leftrightarrow s(\boldsymbol{\kappa}) > \Theta y(1) - \sigma r \forall |\boldsymbol{\kappa}| \geq |\boldsymbol{\kappa}'| \ \textit{ and} \\ W(T=0,\boldsymbol{\kappa},h=0) &< W(T=0,\boldsymbol{\kappa},h=1) \\ &\Leftrightarrow s(\boldsymbol{\kappa}) < \Theta y(0) - \gamma \forall |\boldsymbol{\kappa}| \leq |\boldsymbol{\kappa}''|, \end{aligned}$$

then one equilibrium exists in the unit square $[0,1]^2$. *It is a center.*

Proof. See appendix. □

The public good character of a stabilized economy therefore gives rise to a cyclical behavior of strategy choice by banks and trade unions. The costly long-run strategy does not payoff enough with firm's high survival rates such that it is better for unions to fully use their bargaining power for increased wages. As more and more unions follow this strategy switch, financial investors do not get enough to justify their banking investment and try to raise their financial income by directly investing in firms. However, this destabilizes the economic output path and volatility raises again, pushing unions to reconsider their short-term strategic position in order to switch again. It is therefore justified to speak of an institutional-economic cycle as the characteristics of both the institutional setting and the economic performance vary over the course of the game play.

Finally, getting back to the earlier remark (3.1), it can be shown that the existence of this oscillating behavior depends on the availability of outside labor as stated by the following corollary.

Corollary 4.1. *Provided that* $\{\xi | s(\boldsymbol{\kappa}) > \Theta y(0) - \sigma r - \overline{q}(\xi)(1-\sigma)\} \neq \emptyset \forall \boldsymbol{\kappa}$ *holds, then with increasing availability of outside labor a saddle point emerges and the (**S**, s, FS)-position attracts all flows.*

Proof. See appendix. □

Hence, with increasing immigration (or other forms of outside labor such as a generally higher level of unemployment), the short-term strategy (**S**, *s*, *FS*) will become an attractor and the interior equilibrium will become saddle-point unstable. With increasing availability of labor the amplitude of the cycle dampens and then breaks down when the critical point is reached as can be seen from the simulation (3).

5 Conclusion

The objective of the preceding article has been to show how in a three-player situation with long-term and short-term strategies a strategic complementarity can bring about a coordination game situation with multiple equilibria. In this game, each player – by investing in a relation-specific asset – increases the value of the relation and therefore the marginal incentives for the two remaining players to adopt a similar strategy. With sufficiently high human capital or banking costs, however, the long-term strategy is only profitable if strategy coordination is prevalent.

Moreover, we demonstrated that the resulting coordination equilibria are likely to be unstable – and hence no longer Nash – once one takes the macroeconomic externality into account that is created by the stabilizing effect of the long-term strategy. Abandoning the assumption of local autarky of pools playing, demand is aggregated on the macroeconomic level creating a sectoral correlation of otherwise idiosyncratic shocks. When the resulting collective risk is taken into account – which is justified from the standpoint of these microeconomic considerations – an analysis of the replicator dynamics shows that saddle-point or cyclical equilibria can arise where financial investors and trade unions switch in a cyclical manner between their long- and short-term strategies.

We have argued that this result qualifies in an important way the recent developments of stochastic evolutionary game theory, where shocks are considered to be idiosyncratic to individual players. These games – as well as global games which are isomorphic – select the risk-dominant equilibrium. However, as soon as the underlying stochastic process has to be considered endogenous to the economy – as in our case – one can no longer assume state independency of random shocks, a necessary condition for SGET to yield the risk-dominance result. Under conditions exposed in the last proposition, a Nash equilibrium need not exist even in mixed strategies.

Interesting extensions of the suggested model would be to calibrate a complete macroeconomic model on data concerning the long-run economic and institutional evolution of particular countries and to see whether or not this would reproduce characteristics of their long-run business cycle behavior. One other question which has not been addressed here is the impact of the union's strategic choice on the growth rate. An increasing claim on the national output may reduce incentives to innovative activity and hence reduce growth. A by-product would be a negative relationship between growth and economic volatility, a well-known stylized fact.

6 Appendix

6.1 Proof of proposition 3.1

In order for a coordination game between (**S**, s, FS) and (**L**, l, FL) to exist we must have for financial investors (see game 1):

$$\Psi_1 > \Psi_2 \text{ and } \Psi_3 < \Psi_4 \tag{9}$$

with

$$\Psi_1(T=0,\kappa=0,h) = \delta\pi^e(T=0,\kappa=0)$$

$$= \delta\frac{p(h)y(0)+\overline{s}V^L(T)-w^b}{r+\overline{s}} \tag{10}$$

$$\Psi_2(T=0,\kappa=1,h) = \delta\pi^e(T=0,\kappa=1) - c^B$$

$$= \delta\frac{p(h)y(0)-w^b}{r} - c^B \tag{11}$$

$$\Psi_3(T=1,\kappa=0,h) = \delta\pi^e(T=1,\kappa=0) = \delta\frac{p(h)y(1)-w^b}{r+\overline{s}} \tag{12}$$

$$\Psi_4(T=1,\kappa=1,h) = \delta\pi^e(T=1,\kappa=1) - c^B$$

$$= \delta\frac{p(h)y(1)-w^b}{r} - c^B \tag{13}$$

Substituting (10)–(13) into (9) leads to condition (5).

Given that (5) holds, the conditions for trade unions boil down to the following two inequalities:

$$W_1 > W_2 \text{ and } W_3 < W_4 \tag{14}$$

where $W_1, \dots, W_4$ represent job values under different strategic choices with

$$W_1(T = 0, \kappa = 0, h = 0)$$
$$= \frac{(1-\sigma)(p(0)y(0) + \bar{s}\bar{V})(r + \theta - \bar{q}) + (r\sigma + \bar{s})R}{r[r + \bar{s} + (\theta - \bar{q})(1-\sigma)]} \tag{15}$$

$$W_2(T = 0, \kappa = 0, h = 1)$$
$$= \frac{(1-\sigma)(p(1)y(0) + \bar{s}\bar{V})(r + \theta - \bar{q}) + (r\sigma + \bar{s})(R - \eta)}{r[r + \bar{s} + (\theta - \bar{q})(1-\sigma)]} \tag{16}$$

$$W_3(T = 1, \kappa = 1, h = 0)$$
$$= \frac{(1-\sigma)p(0)y(1)(r + \theta) + \sigma Rr}{r[r + (1-\sigma)\theta]} \tag{17}$$

$$W_4(T = 1, \kappa = 1, h = 1)$$
$$= \frac{(1-\sigma)p(1)y(1)(r + \theta) + \sigma(R - \eta)r}{r[r + (1-\sigma)\theta]} \tag{18}$$

where $\bar{s} = s(0)$, $\bar{q} = q(0)$ and $\bar{V} = V^L(0)$. Substituting (15)–(18) into (14) leads to condition (6). ■

6.2 Proof of propositions 4.1 and 4.2

In order to reduce the complexity of the dynamic model we want to concentrate on the shifts in financial investors' and trade unions' strategy. Firms are supposed to always play the best reply strategy and therefore the kind of technology they adopt on the individual level is not subject to dynamic lags. However, as trade unions and financial investors base their strategic decision on the whole population as in a 'playing the field' model ([10], pp 72–73) they use a statistical model to determine their best reply. Moreover, we will make extensive use of the continuity of the existing probability and the shock with respect to the distribution of strategies (see (8); this is obviously only justified in a large population (large N).

Let us first define the following two variables:

$k : \boldsymbol{\kappa} \mapsto [0, 1]$	percentage of bank-based financed firms
ω	percentage of unions following a wage maximizing strategy.

In the following dynamic analysis we suppose that the growth rate of the number of players using the same pure strategy depends positively on the excess payoff over the average payoff in its player population. Using Taylor's formulation of the n-population replicator dynamics this is written as (see [9]):

$$\dot{x}_{ih} = [u_i(e_i^l, x_{-i}) - u_i(x)]x_{il}$$

with $i \in \{1, .., n\}$, pure strategy $l \in S_i$, S_i: strategy set for player population i and population state x. For the two-player game and with payoff matrix A_i having zeros off the diagonal (like in a coordination game) this can be rewritten as:

$$\dot{x}_{il} = [a_{1i}x_j - a_{2i}(1 - x_j)]x_i(1 - x_i),\ j \neq i.$$

With this simplified form of the replicator dynamics we are then able to prove the propositions in this section.

Opposition between unions and stock markets

Given the assumption about the firm's behavior the game resembles a coordination game between financial investors and workers. As the workers payoff matrix is not supposed to depend on the stochastic distribution (given the strategy choices of the two other players, the workers will have the same best reply strategy independently of $\boldsymbol{\kappa}$) we can represent their payoff matrix as:

$$W = \begin{pmatrix} W_S & 0 \\ 0 & W_L \end{pmatrix}$$

where $W_S \equiv W_1 - W_2$, $W_L \equiv W_4 - W_3$. Columns represent worker's strategy and rows the financial investor's strategies (which corresponds to the firm's technology choice given the assumption about its best replies). The payoff matrix for financial investors is a little different:

$$F = \begin{pmatrix} F_S \cdot (k - k'') & 0 \\ 0 & F_L \cdot (k' - k) \end{pmatrix}$$

where $k' = |\boldsymbol{\kappa}'|$ and $k'' = |\boldsymbol{\kappa}''|$ and $F_S \equiv \Psi_1 - \Psi_2$, $F_L \equiv \Psi_4 - \Psi_3$. Given the continuity of $s(\kappa)$ with respect to κ (see (8)), the two conditions of proposition 4.1 can be translated by giving financial investors a negative return at the short-term equilibrium for $k < k''$ and a negative return for the long-run equilibrium for $k > k'$. For our argument, only the relative payoff evolution is relevant here.

The resulting replicator dynamics is then written as:

$$\dot{k} = [F_L \cdot (k' - k)(1 - \omega) - F_S \cdot (k - k'')\omega] \cdot k(1 - k)$$

$$\dot{\omega} = [W_S \cdot k - W_L \cdot (1 - k)] \cdot \omega(1 - \omega)$$

with isoclines $\dot{k} = 0 \Rightarrow \widetilde{\omega} = [F_L \cdot (k' - k)]/[F_L \cdot (k' - k) + F_S \cdot (k - k'')]$ and $\dot{\omega} = 0 \Rightarrow \widetilde{k} = W_L/[W_S + W_L]$. For $k' < \widetilde{k} < k''$ the two lines cut and an interior stationary point $(\widetilde{k}, \widetilde{\omega})$ exists. The Jacobian for this point is

$$\boldsymbol{J} = \begin{pmatrix} -(F_S \cdot \widetilde{\omega} + F_L(1 - \widetilde{\omega}))\widetilde{k}(1 - \widetilde{k}) & [F_S \cdot (k'' - \widetilde{k}) - F_L \cdot (k' - \widetilde{k})] \cdot \widetilde{k}(1 - \widetilde{k}) \\ (W_S + W_L) \cdot \widetilde{\omega}(1 - \widetilde{\omega}) & 0 \end{pmatrix}$$

with trace $TR = J_{11} < 0$ and determinant $Det = -J_{21}J_{12}$. Moreover we have $J_{21} > 0$, $J_{12} > 0$ for $F_L/F_S > (1 - k'')/(1 - k')$. Provided that $k'' > k'$ and – by definition – $F_L > F_S$, this last condition will be satisfied and hence the interior equilibrium will be saddle-point unstable. ■

Union-led cycles

Under repercussions of financial investors' strategies on workers' best replies the workers' payoff matrix has to be modified. We suggest analyzing the simplest case where investors' best replies do not change due to composition effects; their payoff matrix remains unchanged. When the conditions of the proposition hold, the workers' payoff matrix therefore resembles:

$$W = \begin{pmatrix} W_S(k - k'') & 0 \\ 0 & W_L(k' - k) \end{pmatrix}$$

where $k' = |\boldsymbol{\kappa}'|$ and $k'' = |\boldsymbol{\kappa}''|$ and W_S and W_L are defined as above. Using a similar argument as above, given the continuity of $s(\kappa)$ with respect to κ (see (8)), the two conditions of proposition 4.2 can be translated by giving workers a negative payoff at the short-term equilibrium for $k < k''$ and a negative payoff for the long-run equilibrium for $k > k'$. Again, for our argument, only the relative payoff evolution is relevant here.

The payoff matrix for financial investors is now standard again:

$$F = \begin{pmatrix} F_S & 0 \\ 0 & F_L \end{pmatrix}$$

with F_S and F_L as defined earlier.

The resulting replicator dynamic system is then writen as:

$$\dot{k} = [F_L \cdot (1-\omega) - F_S\omega] \cdot k(1-k)$$
$$\dot{\omega} = [W_S(k-k'')k + W_L(k-k')(1-k)] \cdot \omega(1-\omega)$$

with isoclines

$$\dot{k} = 0 \Rightarrow \widetilde{\omega} = \frac{F_L}{F_S + F_L} \text{ and}$$

$$\dot{\omega} = 0 \Rightarrow \widetilde{k}_{1/2} = \frac{1}{2(W_L - W_s)}$$
$$\left[W_L(1+k') - W_S k'' \pm \sqrt{(W_L(1+k') - W_S k'')^2 - 4(W_L - W_S)W_L k'}\right]$$

As can be easily checked, the upper cutting point $(\widetilde{k}_2, \widetilde{\omega})$ lies outside the unit square for all $W_L > W_S$ and $k'' < k'$. The Jacobian for the remaining equilibrium inside the play grid of the new system now is:

$$\boldsymbol{J}_1 = \begin{pmatrix} 0 & [F_S + F_L] \cdot \widetilde{k}_1(1-\widetilde{k}_1) \\ 2(W_L - W_s)\widetilde{k}_1 - W_L(1+k') + W_S k'' & 0 \end{pmatrix}$$

with trace $TR = 0$ and determinant $Det = -J_{21}J_{12}$. As $J_{12} > 0$ and $J_{21} < 0$, the determinant will be positive leading to two complex eigenvalues. Given that $TR = 0$, the equilibrium will be a center.

With increasing net immigration, ξ, the short-term strategy may become dominant for firms and financial investors, whatever the strategic choice of unions. Using W_1 and W_2 this means $W_1(\overline{\xi}) > W_2(\overline{\xi})\forall\boldsymbol{\kappa}$. Moreover, one can easily show that $W_S(\xi) \equiv W_1(\xi) - W_2(\xi)$ is increasing with ξ. Therefore when $\exists\overline{\xi}$ such that $q(\overline{\xi}) < 1 \wedge W_1(\overline{\xi}) > W_2(\overline{\xi})\forall\boldsymbol{\kappa}$, the above worker payoff matrix changes and rather looks like

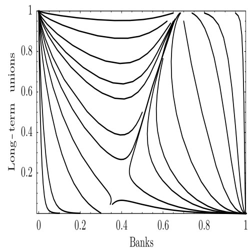
$$W = \begin{pmatrix} W_S & 0 \\ 0 & W_L(k'-k) \end{pmatrix}$$

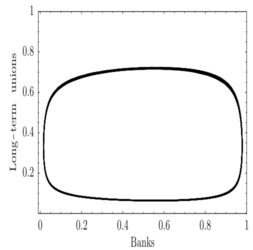
where no $k'' \in [0, 1]$ exists such that $W_1 < W_2$. Carrying out similar calculations as before with this modified payoff matrix, one concludes that the Jacobian for the equilibrium inside the play grid has now a negative determinant, making the equilibrium a saddle point. Moreover, given that the high equilibrium is still unstable for $k > k'$, only the short-run equilibrium will be stable. ■

6.3 Simulations

The simulations are presented in Tables 2 and 3.

Table 2: Banks' and Unions' codynamics





The above simulations have been run using the following system of differential equations:

$$\dot{k} = [F_L \cdot w(t)(k(t) - k'') - F_S(k' - k(t)) \cdot (1 - w(t)) \cdot k(t)(1 - k(t))]$$
$$\dot{w} = [W_L \cdot k(t) - W_S \cdot (1 - k(t))] \cdot w(t)(1 - w(t))$$

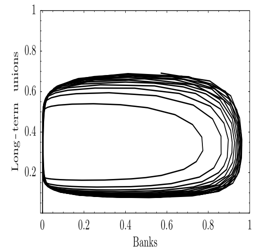
where $W_S = 2$, $W_L = 3$, $F_S = 2$, $F_L = 4$, $k' = 0.3$, $k'' = 0.7$.

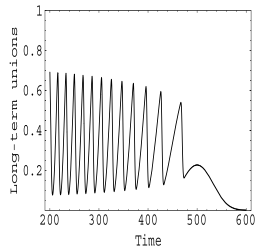
The above simulation has been run using the following system of differential equations:

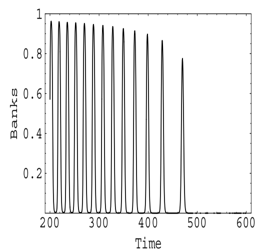
$$\dot{k} = [F_L \cdot w(t) - F_S(1 - w(t))] \cdot k(t)(1 - k(t))$$
$$\dot{w} = [W_L \cdot k(t) - W_S \cdot (1 - k(t))] \cdot w(t)(1 - w(t))$$

where $W_S = 2$, $W_L = 3$, $F_S = 2$, $F_L = 4$.

Table 3: Increasing availability of outside labor







The simulation on this page has been run using the following system of differential equations:

$$\dot{k} = [F_L \cdot w(t) - F_S(1 - w(t))] \cdot k(t)(1 - k(t))$$
$$\dot{w} = [W_L \cdot k(t)(k'' - k(t)) - W_S \cdot (k(t) - k'(1 - 0.002 \cdot t)) (1 - k(t))] \cdot w(t)(1 - w(t))$$

where $W_S = 2$, $W_L = 3$, $F_S = 2$, $F_L = 4$, $k' = 0.3$, $k'' = 0.7$.

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