# Chapter 6

# Monetary and fiscal policy in a growing economy

The discussion of concepts necessary for the analysis of medium-run macroeconomic dynamics in the previous part has led us to consider search and matching as an appropriate means of understanding the restructuring on labour and capital markets. In this first chapter of part II, we now want to put these considerations into practice in a first stylised and deterministic model. In particular, given the non-stochastic nature of the model developped in this chapter, we want to concentrate on the role of monetary and fiscal policy in determining savings and investment behaviour, both independently and in interaction. Similar to the New Keynesian literature around limited participation of households in financial markets (e.g. Lucas (1990), Christiano (1991) and Fuerst (1992)), the chapter concentrates on financial market frictions to understand the real effects of macroeconomic policies.

One simple, alternative way of introducing imperfect financial markets in an otherwise standard macro-model is to consider search frictions that extend to financial markets in a similar fashion as the - by now standard - set-up of search frictions on labour markets. This allows a natural role for financial intermediaries and costly adjustment of capacity depending on the liquidity of financial markets. Such a framework would allow to mimic limited participation as part of the funds are not being funnelled to productive investments but stuck in the matching process, while on the other hand providing some appealing characteristics both methodologically and theoretically.

The methodological advantages of a search-theoretic approach lie in the parametric nature of its rigidities that can be modelled by references to the parameters both of the matching function and of the institutional framework in which the economy operates. In this regard, the matching approach has made strong inroads into labour economies precisely because it adopts a black-box approach that avoids introducing real and nominal rigidities in an ad hoc manner. Hence, methodologically such a generalised search approach would be attractive as it would stay away from mixing parametric and ad hoc forms of rigidities such as those in the current New Keynesian macroeconomics literature.

Theoretically, such an approach could prove fruitful due to the specific set-up of search models: the stock-flow link for each individual decision economic agents make in these models. This stock-flow link is a peculiar feature of the search literature - in contrast to the competitive RBC world where households will invest in only one asset - as every decision (to work, to open a vacancy, to supply funds, to search for a new product) is modelled in terms of an asset that is being accumulated (the expected discounted wealth of a wage contract, the expected net present value of a filled job, etc.) through an investment decision. These models allow for far more different types of liquidity effects than in the standard literature creating new interesting transmission mechanisms of shocks (see for a recent example, Wang and Xie, 2003). In particular, labour supply and savings decisions no longer depend only on current wages, prices and interest rates but on the future expected liquidity of the labour and financial markets.

Based on such a framework with search and matching on financial markets, money demand arises naturally in the model of this paper without any additional assumptions regarding cash-in-advance or the Sidrauskian "money in the utility function" hypotheses. This allows us to formulate straightforwardly an optimal money supply rule against which different monetary policy regimes must be assessed. In addition, the characteristics and determining factors of such a rule can be analysed, in particular the impact of endogenous (AK-)growth, and possible Mundell-Tobin liquidity effects on growth.

Similar to the role of money in the growth process, fiscal policy - in particular the financing side of fiscal policy - can occupy a central role in models where perfect intermediation between deposits and bonds do not exist. Here, we consider a model where government is perfectly social waste but where government activity in form of the financing through taxes or bonds can modify the optimal growth path. Government spending needs to be backed by taxation (at least in the Ricardian set-up that is considered here) but government bonds will provide liquidity to the financial market that raises incentives for households to save and deposit money rather than to consume their income. Hence, there will be an optimal tax-bond equilibrium with non-trivial public debt such as to maximise the optimal growth path.

The chapter starts with the introduction of the necessary modelling concepts to analyse search and matching on both labour and financial markets, based on earlier work by Pissarides (2000), Becsi, Li and Wang (2000), Den Han, Ramey and Watson (1999), Scott and Herny (2003) and Wasmer and Weil (2004). Introducing intertemporally optimising households and firms, the financial and labour market equilibria are determined, first in a stationary model without growth and then in an endogenous growth model based on a simple AK-growth mechanism. Section 5 looks at the impact of both structural and macroeconomic policies. In particular, it is shown that due to the ambiguous role played by the inflation rate, an optimal, positive inflation rate is shown to exist that balances the direct negative impact of inflation on growth with its positive impact on financial market liquidity through the household's portfolio decisions. A similar effect can be shown to exist for tax-financed public debt, albeit being limited to a particular sub-set of financial market equilibria. Finally, a positive interaction between monetary and fiscal policy can be demonstrated.

# 6.1 The macroeconomy

Three types of agents are considered: entrepreneurs, workers and financiers. Entrepreneurs have ideas but cannot work in production and possess no capital. Worker transform entrepreneurs' ideas into output but have neither entrepreneurial skills nor capital; financiers (or bankers) have access to the financial resources required to implement production but cannot be entrepreneurs nor workers. A productive firm is thus a relationship between an entrepreneur, a financier and a worker. In the following, decentralised economy, however, only the optimal program of firms and households are considered, the activitiy of financial investors in intermediating deposits into bonds is assumed to be summarised in a matching function of the financial market.

## 6.1.1 Financial and labour market search

Labor market frictions are present under the form of a matching process à la Pissarides (2000), with a constant returns matching function[1] $m^L(s \cdot \mathcal{U}, \mathcal{V})$. Matches between workers and firms depend on job vacancies $\mathcal{V}$, unemployed workers $\mathcal{U}$ and workers' search intensity $s$. From the point of view of the firms, labor market tightness is measured by $\theta \equiv \mathcal{V}/\mathcal{U}$. Labor market liquidity will be $1/\theta$. The instantaneous probability of finding a worker is thus $m^L(s \cdot \mathcal{U}, \mathcal{V})/\mathcal{V} = m^L(s/\theta, 1) \equiv q(\theta, s)$, $q_\theta(\theta, s) < 0$ while for the worker the instantaneous probability to find a firm writes as $m^L(s \cdot \mathcal{U}, \mathcal{V})/\mathcal{U} = m^L(s, \theta) \equiv \theta q(\theta, s)$, $\frac{\partial(\theta q(\theta,s))}{\partial \theta} > 0$. Moreover, we have $q_s(\theta, s) > 0$. Finally, the aggregate matching process yielding $N$ filled jobs, and normalising the labour force to unity, i.e. $L = 1$, we have $\mathcal{U} = L - N \Rightarrow u^l \equiv \frac{L-N}{L} = 1 - n$, with $n \equiv \frac{N}{L}$.

Similarly, increasing its capital stock, a firm needs to find a financing source (all finance is supposed to be external) on financial markets, which are subject to search frictions as well. In parallel to labour market search, matching on financial market is characterised by the constant returns matching function $m^B(\mathcal{D}, \mathcal{B})$ where $\mathcal{D}$ is the deposit amount by households to be intermediated into entreprises' bonds $\mathcal{B}$ to finance an increase of their capital stock. From the point of view of firms, credit market tightness is measured by $\phi \equiv \mathcal{B}/\mathcal{D}$ and $1/\phi$ is an index of credit market liquidity, i.e. the ease with which entrepreneurs can find financing. The instantaneous probability that an entrepreneur will find a banker is $m^B(\mathcal{D}, \mathcal{B})/\mathcal{B} = m^B(1/\phi, 1) \equiv p(\phi)$. This probability is increasing in credit market liquidity, i.e. $p'(\phi) < 0$.

## 6.1.2 The household's optimal problem

Households' wealth is hold either in pyhsical capital, government bonds or money. The economy's national accounts, therefore, write (in real terms) as:

[1] $m^L$ has positive and decreasing marginal returns on each input.

| **Assets** | **Liabilities** |
|---|---|
| K: Physical capital | W: Households' wealth |
| B: Government bonds | |
| M: Real money balances | |

Money demand arises endogenously in this model as search friction on the financial market put a wedge between households' deposits, $D$, and their transformation into interest bearing titles (either equity, $E$, or government bonds, $B$). Hence, an "unemployment rate" of unmatched financial assets can be defined as: $u^f = \frac{W-(K+B)}{W} = \frac{M}{W}$. Households hold part of ther financial wealth ($b$) in government bonds, the rest is invested in equity ($(1-b)$), i.e. $K = (1-b)\left(1-u^f\right)W$ and $B = b\left(1-u^f\right)W$.

Households earn different returns on the three assets, denoted as $r_K$, $r_B$ and $r_M$. Real money balances do not earn interest but depreciate with the inflation rate, $\pi$, hence $r_M = -\pi$. Moreover, households have to pay taxes on capital income (supposed to apply only on income earned from physical capital investment, not on government bonds), hence the after tax rate of return on capital holdings write as $\widetilde{r}_K = (1-\tau_K)\, r_K$, with $\tau_K$: taxes on capital income. Finally, in the absence of stochastic returns, the equilibrium condition for households to hold either bonds or financial assets in non-trivial amounts requires that the after-tax return on investment on each of the two assets are equal, i.e. $\widetilde{r}_K = r_B \Leftrightarrow r_K\,(1-\tau_K) = r_B$.

Households acccumulate wealth through deposits $\mathcal{D}_t = Y - c$ with income generated from asset returns and gainful employment $Y = r_W W + (1-\tau_w)\, wN$, with $\tau_w$: taxes on labour income, but have to spend search costs on labour markets[2], assumed to be proportional to their expected net wage and the labour market tightness, $1-N$:

$$\begin{aligned}\dot{W} \;=\; & \left\{\left[r_K\,(1-\tau_K)\,(1-b) + r_B b\right]\left(1-u^f\right) - \pi u^f\right\} W + (1-\tau_w)\, wN \\ & -c - w\Phi\,(s)\,(1-N) \end{aligned} \tag{6.1}$$

Besides wealth, households decide on their search effort, $s$, on labour markets to find a suitable job, thereby taking the evolution of employment into account, which is given by:

$$\dot{N} = m^L\,(s,\theta)\,(1-N) - \sigma N \tag{6.2}$$

where $\sigma$: the rate of job destruction assumed to be exogenously given.

Households maximise intertemporal utility, depening on consumption, $c_t$, and employment, $N_t$ choosing consumption, labour market search effort and deposits:

$$\max_{c_t, s_t, \mathcal{D}_t} \int_0^\infty U\,(c_t, N_t)\, e^{-\rho t}$$

with $c_t$: the household's consumption decision, $s_t$: the household's search intensity for a job, $\mathcal{D}_t = Y - c$ the amount of (new) deposits[3] and $\rho$ the (constant) intertemporal time preference rate.

[2]Search costs are opportunity costs in terms of *utility* forgone.

[3]There is a possibility to introduce a deposit emission cost, $\xi$, proportional to the amount of free deposits and similar to the bank entry costs in Wasmer and Weil's model. However, nothing substantial is changed, the cost entering only the labour market determination.

Assuming a CRRA contemporaneous utility function with risk aversion $\eta$ and leisure weight $\chi < 1$, the Hamiltonian writes as[4]:

$$\begin{aligned}\mathcal{H} &= \frac{c^{1-\eta}}{1-\eta} - \chi \log(N) \\ &\quad + \nu_1 \left\{ [r_K (1-\tau_K)(1-b) + r_B b] \left(1-u^f\right) W - \pi u^f W - c \right. \\ &\quad \left. + (1-\tau_w) wN - w\Phi(s)(1-N) \right\} \\ &\quad + \nu_2 \left[ m^L(s,\theta)(1-N) - \sigma N \right].\end{aligned}$$

The household's first-order conditions can then be determined as:

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \Leftrightarrow c^{-\eta} = \nu_1 \tag{6.3a}$$

$$\frac{\partial \mathcal{H}}{\partial s} = 0 \Leftrightarrow \nu_1 w \Phi'(s) = \nu_2 m_s^L(s,\theta) \tag{6.3b}$$

The co-state variables $\mathcal{B}$ and $n$ evolve according to:

$$\dot{\nu}_1 = \rho\nu_1 - \frac{\partial \mathcal{H}}{\partial W} = \rho\nu_1 - \nu_1 \left\{ [r_K(1-\tau_K)(1-b) + r_B b] \left(1-u^f\right) - \pi u^f \right\} \tag{6.4a}$$

$$\dot{\nu}_2 = \rho\nu_2 - \frac{\partial \mathcal{H}}{\partial N} = \rho\nu_2 + \frac{\chi}{N} - \nu_1 w (\Phi(s) + 1 - \tau_w) + \nu_2 \left(\sigma + m^L(s,\theta)\right) \tag{6.4b}$$

Deriving (6.3a) with respect to time and substituting $\dot{\nu}_1$ and $\nu_1$ in (6.4a), the household's optimal consumption path can be determined as:

$$-\eta \dot{c} c^{-\eta-1} = c^{-\eta} \left(\rho - r_K(1-\tau_K)\left(1-u^f\right) + \pi u^f\right)$$

$$\Leftrightarrow \hat{c} \equiv \frac{\dot{c}}{c} = \frac{1}{\eta}\left(r_K(1-\tau_K)\left(1-u^f\right) - \pi u^f - \rho\right) \tag{6.5}$$

[RECALCULATE] Labour supply - determined indirectly through the search effort $s$ - can then be determined similarly by substituting out $\nu_1$ in (6.3b) and (6.4b) and taking derivatives of (6.3b) with respect to time. In steady state, the optimal search effort $s^*$ has therefore to satisfy the following equation (see Appendix 8.3):

$$\frac{\Phi'(s^*)}{\Phi(s^*)} = \frac{m_s^L(s^*,\theta)}{\rho + \sigma + m^L(s^*,\theta) - (1-\eta) g}.$$

Now, suppose $\Phi(s) = \exp(a_0 s)$ hence $\Phi_s(s) = a_0 \exp(a_0 s)$ and $m^L(s,\theta) = s \cdot \theta q(\theta)$, hence

$$s^* = \frac{g(1-\eta) - (\rho+\sigma)}{\theta q(\theta)} + \frac{1}{a_0}.$$

[4] Alternatively the contemporaneous utility function:

$$\frac{c^{1-\eta}}{1-\eta} \cdot \frac{\chi}{\log(n)}$$

could have been used which would lead to non-separability of the employment and consumption decision. However, the characteristics of employment lotteries would have been lost (see Merz, 1995).

Alternatively, suppose $\Phi(s) = a_0 s^{a_1}$ hence $\Phi_s(s) = a_0 a_1 s^{a_1 - 1}$ and $m^L(s, \theta) = s^{\alpha} \cdot \theta q(\theta)$, hence

$$s^* = \left( \frac{a_1 \left( g \left( 1 - \eta \right) - \left( \rho + \sigma \right) \right)}{\theta q(\theta) \left( a_1 - \alpha \right)} \right)^{\frac{1}{\alpha}}$$

These two optimality conditions lead to the following proposition:

**Proposition 1** *In steady state, growth increases with the rate of return on capital and decreases with capital taxes, inflation and unmatched financial resources. Moreover, search effort decreases with income taxation, but increases with wages, labour market tightness and total employment.*

## 6.2 Search and matching

### 6.2.1 Firms

In order to determine the equilibrium on financial and labour markets, we need to turn, now, to the optimal program of the firm. In the following, we normalise labour supply to unity and assume completely elastic supply of capital with interest rates fixed on world capital markets. In the production process, jobs disappear with rate $\sigma$ and capital depreciates at rate $\delta$. Assuming labour-saving technological progress, $A$, the firm's constant-returns-to-scale production function writes as:

$$F = F(K, AN), \;\; F_K > 0, F_N > 0, F_{KK} < 0, F_{NN} < 0.$$

The firm maximises profits selecting vacancies, $\mathcal{V} \geq 0$, and issues of bonds, $\mathcal{B} \geq 0$, subject to costs of job vacancy creation, $c \cdot w\mathcal{V}$, and finding a financial investor, $\kappa \cdot r\mathcal{B}$. Accordingly, its optimal program writes as:

$$\max_{\mathcal{V},\mathcal{B}} \int_0^{\infty} \nu_1(t) \left( F(K, AN) - wN - rK - \zeta \cdot w\mathcal{V} - \kappa \cdot r\mathcal{B} \right) e^{-\rho t} dt$$

where $\rho$ stands for the intertemporal preference rate and $\nu_1(t)$ for the household's shadow variable related to its accumulation constraint[5]. Hence, using the dynamics of employment and capital growth:

$$\begin{aligned} \dot{N} &= q(s, \theta) \mathcal{V} - \sigma N \\ \dot{K} &= p(\phi) \mathcal{B} - \delta K \end{aligned}$$

the profit maximisation problem can be rewritten as[6]

$$\max_{\mathcal{V},\mathcal{B}} \int_0^{\infty} \nu_1(t) \left( F(K, AN) - wN - rK - \zeta \cdot w \frac{\dot{N} + \sigma N}{q(\theta)} - \kappa \cdot r \frac{\dot{K} + \delta K}{p(\phi)} \right) e^{-\rho t} dt \qquad (6.6)$$

[5] This follows the spirit of Merz (1995) determination of the competitive search equilibrium (see Merz, 1995, pp. 287-293).

[6] See appendix 1 for the solution of the optimal program by way of a Hamiltonian.

Wages and interest rates are negociated on labour and financial markets respectively. Workers enjoy bargaining power $\beta$, financial investors pressure for interest rates with bargaining power $\gamma$. In order to simplify the bargaining process, we will follow Eriksson (1997), here assuming that the bargaining power describes the bounds between which wages and interest rates have to fall:

- Wages will be determined as a weighted average of marginal productivity, $F_N$, and the gain of filling a vacancy on the one hand, $\theta\zeta_0$, and unemployment benefits, $R_0$, on the other:
$$w = \beta\left(F_N + \theta\zeta_0\right) + (1-\beta) R_0.$$
- Interest payment will be determined as a part of the marginal productivity of capital and the gain of matching issued debt, assuming that the fall-back option for financial investors is zero (we are assuming a closed economy here):
$$r = \gamma\left(F_K - \delta + \phi\kappa_0\right).$$

Then, capturing the increase in productivity, which will affect the increase in the reservation wage, the vacancy costs and the costs of scheduling new debt over time, we write $R_0 = R \cdot w$ $(0 \leq R < 1)$, $\zeta_0 = \zeta \cdot w$ and $\kappa_0 = \kappa \cdot r$, and can therefore reformulate the two prices as:

$$\begin{aligned} w &= \beta\left(F_N + \theta\zeta \cdot w\right) + (1-\beta) R \cdot w \Leftrightarrow w = \frac{\beta}{1 - \beta\theta\zeta - (1-\beta) R} F_N && (6.7) \\ r &= \gamma\left(F_K - \delta + \phi\kappa \cdot r\right) \Leftrightarrow r = \frac{\gamma}{1 - \gamma\phi\kappa}\left(F_K - \delta\right) && (6.8) \end{aligned}$$

Taking the labour and financial market liquidities as given, the first-order conditions of the firm's problem can then be written as:

$$\begin{aligned} \nu_1(t)\left(F_N - w - \frac{\zeta \cdot w\sigma}{q(\theta)}\right) e^{-\widetilde{\rho}(t)t} - \frac{\partial}{\partial t}\left(-e^{-\rho t}\nu_1(t)\frac{\zeta w}{q(\theta)}\right) &= 0 \\ \nu_1(t)\left(F_K - r - \frac{\kappa \cdot r\delta}{p(\phi)}\right) e^{-\widetilde{\rho}(t)t} - \frac{\partial}{\partial t}\left(-e^{-\rho t}\nu_1(t)\frac{\kappa r}{p(\phi)}\right) &= 0 \end{aligned}$$

or alternatively:

$$\begin{aligned} \nu_1(t)\left[F_N - w + \zeta w\frac{\widehat{w} - \rho - \sigma}{q(\theta)}\right] + \dot{\nu}_1(t)\frac{\zeta \cdot w}{q(\theta)} &= 0 \\ \nu_1(t)\left[F_K - r + \kappa r\frac{\widehat{r} - \rho - \delta}{p(\phi)}\right] + \dot{\nu}_1(t)\frac{\kappa \cdot r}{p(\phi)} &= 0 \end{aligned}$$

where $\widehat{w} = \frac{\dot{w}}{w}$ and $\widehat{r} = \frac{\dot{r}}{r}$. Here, the first conditions describes the job creation condition, while the second determines the capital accumulation condition. As in the single-matching case (with matching only on the labour market), the equilibrium can then be determined by way of reference to the wage and interest curves, (6.7) and (6.8). Moreover, the unused reserves on labour and financial markets, $u^l$ and $u^f$, can be determined, by way of equalizing inflows and outflows on both markets:

1. On financial markets, in each period $u^f \cdot \phi p(\phi)$ gets matched, but $\left(1-u^f\right)\delta$ quit the market, where $u^f$ as defined above. Hence, in equilibrium:

$$\begin{aligned}\left(1-u^f\right)\delta &= u^f \cdot \phi p(\phi)\\ u^f &= \frac{\delta}{\delta+\phi p(\phi)}\end{aligned}$$

2. On labour markets, in each period $u^l \cdot \theta q(\theta)$ gets matched, but $\left(1-u^l\right)\sigma$ quit the market. Hence, in equilibrium:

$$\begin{aligned}\left(1-u^l\right)\sigma &= u^l \cdot \theta q(\theta)\\ u^l &= \frac{\sigma}{\sigma+\theta q(\theta)}\end{aligned}$$

Denoting $k = K/(AN)$ and $f(k) = \frac{1}{AN}F(K, AN)$, the firm's first-order conditions can be rewritten in intensive form as:

$$\begin{aligned}\nu_1(t)\left[(f(k)-kf'(k)) - w - \zeta\cdot w\frac{\sigma+\rho-\widehat{w}}{q(\theta)}\right] + \dot{\nu}_1(t)\frac{\zeta\cdot w}{q(\theta)} &= 0\\ \nu_1(t)\left[f'(k) - r - \kappa\cdot r\frac{\delta+\rho-\widehat{r}}{p(\phi)}\right] + \dot{\nu}_1(t)\frac{\kappa\cdot r}{p(\phi)} &= 0\end{aligned}$$

and the wage and interest curve write as:

$$w = \frac{\beta}{1-\beta\theta\zeta-(1-\beta)R}(f(k)-kf'(k)) \tag{6.9}$$

$$r = \frac{\gamma}{1-\gamma\phi\kappa}f'(k) \tag{6.10}$$

This can be used to determine the steady-state equilibrium $(\theta,\phi)$ where $\widehat{r}=0$ and $\nu_1(t) = c^{-\eta} \Rightarrow \frac{\dot{\nu}_1(t)}{\nu_1(t)} = -\eta\frac{\dot{c}}{c}$:

$$(1-\beta)(1-R) - \beta\theta\zeta - \frac{\zeta\beta}{q(s,\theta)}\left(\sigma+\rho-\widehat{w}+\eta\frac{\dot{c}}{c}\right) = 0 \tag{6.11}$$

$$1-\gamma(1+\phi\kappa) - \frac{\kappa\gamma}{p(\phi)}\cdot\left(\delta+\rho+\eta\frac{\dot{c}}{c}\right) = 0 \tag{6.12}$$

relating $\theta$ and $\phi$ to the model's fundamentals. Moreover, under the assumption that interest rates are fixed on world markets, equation (6.9) will also allow to determine the optimal capital-labour intensity.

### 6.2.2 Steady state in the no-growth equilibrium

When long-term per capita growth is zero, we have $\frac{\dot{c}}{c} = \widehat{w} = 0$ and $A = A_0 = \text{const.}$ Following Merz (1995) and Shi and Wen (1997), we want to assume $\Phi' = 0$ and $s^* = s_0 = \text{const}$. From (**??**), (6.5), (6.9), (6.10), (6.11), (6.13b), the steady state values of the vector $\{\phi, r, \theta, k, n, w, c\}$ can hence be determined recursively as (where $u^f = u^f(\phi^*)$ and $u^l = u^l(\theta^*)$):

$$
\begin{aligned}
\phi^* &: \quad 1-\gamma(1+\phi^*\kappa) - \frac{\kappa\gamma}{p(\phi^*)}\cdot(\delta+\rho) = 0 \\
r^* &= \quad \frac{\rho}{1-u^f} \\
\theta^* &: \quad (1-\beta)(1-R) - \beta\theta^*\zeta - \frac{\zeta\beta}{q(s^*,\theta^*)}(\sigma+\rho) = 0 \\
k^* &: \quad f'(k^*) = \frac{1-\gamma\phi^*\kappa}{\gamma} r^* \\
N^* &: \quad N^* = \frac{m^L(s^*,\theta^*)}{\sigma + m^L(s^*,\theta^*)} \Rightarrow K^* = A_0 k^* N^* \\
w^* &: \quad w^* = \frac{\beta}{1-\beta\theta^*\zeta - (1-\beta)R}\left(f(k^*) - k^* f'(k^*)\right) \Rightarrow Y^* = w^* N^*
\end{aligned}
$$

Note that it can be shown that with constant-returns-to-scale matching functions $m^b$ and $m^l$, $\phi^*$ and $\theta^*$ are unique solutions to their steady-state equations.

Moreover we have:

$$\dot{K} = p(\phi)\mathcal{B} - \delta K = 0 \Rightarrow \mathcal{B}^* = \frac{\delta K^*}{p(\phi^*)}$$

which yields, together with $\dot{\mathcal{B}} = 0$ and $K^* = \left(1-u^f\right)a^*$:

$$
\begin{aligned}
c^* &= r^*K^* + Y^* - w^*\Phi(s^*)(1-N^*) - \zeta w^*\theta^* u^l - \kappa r^*\mathcal{B}^* \\
&= r^*K^*\left(1 - \frac{\kappa\delta}{p(\phi^*)}\right) + w^*\left[N^*(1+\Phi(s^*)+\zeta\theta^*) - \Phi(s^*) - \zeta\theta^*\right].
\end{aligned}
$$

# 6.3 Growth and matching

## 6.3.1 Exogenous growth and matching

Once the optimal paths for consumption and labour supply determined, the availability of financial funds from which firms can draw their resources derives. This can be used to determine the optimal capital-labour intensity, $k$, that has been left undetermined in the model above. Let multi-factor productivity grow at a constant rate $g$, i.e. $A(t) = A_0 e^{gt}$. In equilibrium, given a constant population, i.e. $\dot{L} = 0$, we have:

$$\frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{\dot{\mathcal{B}}}{\mathcal{B}} = \frac{\dot{A}}{A} = g$$

Recall the optimal consumption path from above:

$$\frac{\dot{c}}{c} = \frac{1}{\eta}\left(r\left(1-u^f\right) - \rho\right)$$

leading to the steady state interest rate:

$$r^* = \frac{\eta g + \rho}{1 - u^f}$$

where $\phi^*$ is determined by (6.13b) below.

Given the exogenously given growth rate of consumption, $g$, together with the interest-rate curve (6.10), this can be used to define the steady-state capital intensity:

$$\begin{aligned} r &= \frac{\eta g + \rho}{1 - u^f} \stackrel{!}{=} \frac{\gamma}{1 - \gamma \phi \kappa} f'(k) \\ &\Rightarrow k^* = f'^{-1}\left[\frac{(\eta g + \rho)(1 - \gamma \phi^* \kappa)}{\gamma (1 - u^f)}\right]. \end{aligned}$$

Moreover, given the positive growth rate and noting the wages will grow at the same rate, the labour and financial market liquidities can be solved by:

$$(1-\beta)(1-R) - \beta\theta^*\zeta - \frac{\zeta\beta}{q(s,\theta^*)}(\sigma + \rho - (1-\eta)g) = 0 \qquad (6.13a)$$

$$1 - \gamma(1 + \phi^*\kappa) - \frac{\kappa\gamma}{p(\phi^*)} \cdot (\delta + \rho + \eta g) = 0. \qquad (6.13b)$$

These steady-state liquidities can be used to derive employment and wages in equilibrium:

$$\begin{aligned} N^* &= \frac{m^L(s^*, \theta^*)}{\sigma + m^L(s^*, \theta^*)} \Rightarrow K^* = A_t k^* N^* \\ w^* &= \frac{\beta}{1 - \beta\theta^*\zeta - (1-\beta) R}\left(f(k^*) - k^* f'(k^*)\right). \end{aligned}$$

Finally, given that $\dot{K}/K = g \Rightarrow p(\phi)\mathcal{B}/K - \delta = g$ we have:

$$\frac{\mathcal{B}^*}{K^*} = \frac{g + \delta}{p(\phi)}.$$

Interestingly, in the exogenous growth case, there is financial market dominance as the steady state liquidity on financial markets "runs the show".

### 6.3.2 Endogenous growth with knowledge spillovers

In order to introduce endogenous growth in its simplest form, we will follow Romer (1986) and consider spillovers from installed capital to the overall economy (scaled by the size of the economy[7]). Hence, the production function for each single firm $i$ writes as:

$$F(K_i, A_i N_i) = A_i \cdot \overline{K}^{\alpha} N_i^{\alpha} K_i^{1-\alpha}$$

[7] The scaling factor is introduced to avoid that the growth rate rises with the size of the economy (Jones, 1995).

where $\overline{K} = N^{-a} \sum K_i$. Hence, in equilibrium where $K_i = K_j$, $N_i = N_j$, $i \neq j$, and therefore $\overline{K} = \frac{N^{1-\alpha}}{N_i} K_i$ with $N$: total employment, the rate of return on capital writes as:

$$r_K = \gamma \left( F_K - \delta + \phi\kappa_0 \right) \Leftrightarrow r_K = \frac{\gamma}{1 - \gamma\phi\kappa} \left( A \left( 1 - \alpha \right) - \delta \right) = \text{const}\,. \tag{6.14}$$

Moreover, from (6.5) we know the steady state growth rate to be:

$$\frac{\dot{c}}{c} = \frac{1}{\eta} \left( r_K \left( 1 - \tau_K \right) \left( 1 - u^f \right) - \pi u^f - \rho \right)$$

which can be rewritten as:

$$\begin{aligned} g = \frac{\dot{c}}{c} &= \frac{1}{\eta} \left( r_K \left( 1 - \tau_K \right) \left( 1 - u^f \right) - \pi u^f - \rho \right) \qquad (6.15) \\ &= \frac{1}{\eta} \left( \frac{\gamma \left( 1 - \tau_K \right) \left( 1 - u^f \right)}{1 - \gamma\phi\kappa} \left( A \left( 1 - \alpha \right) - \delta \right) - \pi u^f - \rho \right) \\ &= \frac{\gamma \left( 1 - \tau_K \right) \left( 1 - u^f \right)}{\eta \left( 1 - \gamma\phi\kappa \right)} \left( A \left( 1 - \alpha \right) - \delta \right) - \frac{\rho + \pi u^f}{\eta} \end{aligned}$$

Moreover, financial and labour market equilibrium is determined from (6.11) and (6.12) as:

$$\begin{aligned} &\begin{cases} \left( 1 - \beta \right) \left( 1 - R \right) - \beta\theta\zeta - \frac{\zeta\beta}{q(s,\theta)} \left( \sigma + \rho + \frac{\eta - 1}{\eta} \left( r_K \left( 1 - \tau_K \right) \left( 1 - u^f \right) - \pi u^f - \rho \right) \right) = 0 \\ 1 - \gamma \left( 1 + \phi\kappa \right) - \frac{\kappa\gamma}{p(\phi)} \cdot \left( \delta + \rho + r_K \left( 1 - \tau_K \right) \left( 1 - u^f \right) - \pi u^f - \rho \right) = 0 \end{cases} \\ \Leftrightarrow &\begin{cases} \frac{\eta}{\eta - 1} \Theta \left( \theta, \beta, \zeta, R \right) - \gamma \left( A \left( 1 - \alpha \right) - \delta + \phi\kappa \right) \left( 1 - \tau_K \right) \left( 1 - u^f \right) = \sigma + \frac{\rho}{\eta} - \pi u^f \\ \Phi \left( \phi, \gamma, \kappa \right) - \gamma \left( A \left( 1 - \alpha \right) - \delta + \phi\kappa \right) \left( 1 - \tau_K \right) \left( 1 - u^f \right) = \delta - \pi u^f \end{cases} \end{aligned} \tag{6.16}$$

where $\Theta \left( \theta, \beta, \zeta, R \right) = \eta q \left( s, \theta \right) \frac{(1-\beta)(1-R) - \beta\theta\zeta}{\zeta\beta(\eta - 1)}$, $\Theta_\theta < 0$, $\Theta_\beta < 0$, $\Theta_R < 0$, $\Theta_\zeta < 0$ and $\Phi \left( \phi, \gamma, \kappa \right)$, $\Phi_\phi < 0$, $\Phi_\gamma < 0$, $\Phi_\kappa < 0$.

**Proposition 2** *The growth rate increases unambiguously with both optimal financial market liquidity $\phi^*$. Moreover, financial and labour market liquidity will be determined simultaneously by the system (6.16). In this regard, higher productivity, $A$, increases (decreases) unemployment for $\eta > 1$ ($\eta < 1$).*

**Proof.** The equilibrium interest rate increases linearly with $\phi$ at slope $\gamma\kappa$. The growth rate, in turn, is positively affected by an increase in the interest rate and a decrease in the amount of unsed funds that suffer from the inflation tax:

$$\frac{dg}{d\phi} = \frac{1}{\eta} \left[ \left( 1 - \tau_K \right) \left( 1 - u^f \right) \frac{dr_K}{d\phi} - \left( r_K \left( 1 - \tau_K \right) + \pi \right) \frac{du^f}{d\phi} \right]. \blacksquare$$

## 6.4 Macroeconomic policies

### 6.4.1 Structural policies

Given real rigidities on labour and financial markets, policy makers can influence the steady state growth rate by modifying the entry costs for issuing bonds or opening vacancies ($\kappa$, $\zeta$) or by influencing the relative bargaining power on financial and labour markets ($\gamma$, $\beta$). Applying Cramer's rule on the system set up by total differencing (6.11), (6.12), (6.14) and (6.15), the following table ensues, summarising the effect of these four parameters on steady state liquidity of financial and labour markets and the impact on growth in equilibrium

| | $\kappa$ | $\zeta$ | $\gamma$ | $\beta$ |
|---|---|---|---|---|
| $\theta$ | $> 0$ for $\eta > 1$<br>$< 0$ for $\eta < 1$ | $< 0$ | $< 0$ for $\eta > 1$<br>$> 0$ for $\eta < 1$ | $< 0$ |
| $\phi$ | $< 0$ | $= 0$ | $< 0$ | $= 0$ |
| $r$ | $< 0$ for $\phi > \underline{\phi}$<br>$> 0$ for $\phi < \underline{\phi}$ | $= 0$ | $< 0$ for $\phi > \underline{\underline{\phi}}$<br>$> 0$ for $\phi < \underline{\underline{\phi}}$ | $= 0$ |
| $g$ | $< 0$ for $\phi > \underline{\phi}$<br>$> 0$ for $\phi < \underline{\phi}$ | $= 0$ | $< 0$ for $\phi > \underline{\underline{\underline{\phi}}}$<br>$> 0$ for $\phi < \underline{\underline{\underline{\phi}}}$ | $= 0$ |

As already mentionned before, the system is characterised by a triangular structure between financial and labour market liquidity: While financial markets affect labour markets, the reverse is not true. This result is a direct consequence of our assumption regarding the sources of endogenous growth in the production function: In a more sophisticated model, growth effects from frictions on labour markets that affect returns to human capital investment may be considered. Notice, morever, that despite an unambiguously negative sign of the financial market sturctural parameters $\kappa$ and $\gamma$ on financial market liquidity, their growth effect are ambiguous and may be positive for low financial market liquidity. This results from the fact that besides lowering steady state financial market liquidity, they also impact upon the rate of return on capital - in an ambiguous way.

### 6.4.2 Monetary policy

In the case of exogenously given growth, inflation will have no impact on the the rate of accumulation of household wealth. However, both in the case of exogenous and endogenous growth, the depreciation of real money balances will impact on the portfolio balance through the financial market liquidity. In particular, financial market tightness will increase as households prefer to withdraw their funds and to redirect them towards consumption[8]. This follows directly from the equilibrium condition for financial market liquidity (6.12):

$$\begin{aligned} 1-\gamma\left(1+\phi\kappa\right)-\frac{\kappa\gamma}{p\left(\phi\right)}\cdot\left(\delta+\rho+\eta\frac{\dot{c}}{c}\right) &= 0 \\ \Leftrightarrow 1-\gamma\left(1+\phi\kappa\right)-\frac{\kappa\gamma}{p\left(\phi\right)}\cdot\left(\delta+r_K\left(1-u^f\right)-\pi u^f\right) &= 0. \end{aligned} \tag{6.17}$$

[8] Recall the money demand arises due to search frictions on *financial* markets not on product markets.

In the exogenous growth model, this does not have any further consequences. However, in the endogenous growth model, a reduction in the financial market liquidity will lead to a rise in the equilibrium before-tax interest rate; this causes the steady state growth rate to increase. Hence the following proposition can be proven:

**Proposition 3** *In the exogenous growth case, there is complete dichotomy between growth rates and the inflation rate (not necessarily between levels and inflation). In the endogenous growth case, however, for $\pi < \overline{\pi}$ - with $\overline{\pi} > 0$ - the steady state growth rate increases with the inflation rate but decreases for $\pi > \overline{\pi}$, leading to a hump shaped impact of inflation on growth, with $\overline{\pi}$ the growth-optimising inflation rate. At this growth-maximising inflation rate, employment is minimised (maximised) for $\eta > 1$ ($\eta < 1$).*

**Proof.** As seen from (6.15), inflation has a first-order negative impact on the growth rate: $\frac{\partial g}{\partial \pi} = -\frac{u^f}{\eta}$ which is independent from the inflation rate. However, inflation also affects financial market liquidity, as an increase in inflation leads to a portfolio shift by households away from savings towards consumption; totally differencing the financial market equilibrium condition (6.17) yields (note that in the following equations $u^f \equiv u^f(\phi^*)$ and $r_K \equiv r_K(\phi^*)$):

$$\frac{d\phi^*}{d\pi} = \frac{p(\phi^*)\,u^f}{p(\phi^*)\left(p(\phi^*) + (1-u^f)\,r_K' - (r_K+\pi)\,u^{f\prime}\right) - p'(\phi^*)\left(\delta + r_K(1-u^f) - \pi u^f\right)} > 0$$

As shown before in proposition 2, both the equilibrium interest rate and the growth rate increase with financial market tightness as less funds are left unused in the household's deposits ($u^f$ decreases with $\phi$). Hence, inflation has a positive second-order impact on the growth rate *via* its impact on the financial market liquidity.

Combining the impact of inflation on financial market liquidity and financial market liquidity on growth, the second-order positive effect writes as:

$$\begin{aligned}
\frac{\partial g}{\partial \phi}\cdot\frac{\partial \phi}{\partial \pi} &= \frac{p(\phi^*)\,u^f\left(\left(1-u^f\right)r_K' - r_K u^{f\prime}\right)}{\eta\left(p(\phi^*)\left(p(\phi^*) + (1-u^f)\,r_K' - (r_K+\pi)\,u^{f\prime}\right) - p'(\phi^*)\left(\delta + r_K(1-u^f) - \pi u^f\right)\right)} \\
&= \frac{u^f(\phi)}{\eta}\,\frac{p(\phi^*)\,u^f\left(\left(1-u^f\right)r_K' - r_K u^{f\prime}\right)}{p(\phi^*)\left(p(\phi^*) + (1-u^f)\,r_K' - (r_K+\pi)\,u^{f\prime}\right) - p'(\phi^*)\left(\delta + r_K(1-u^f) - \pi u^f\right)}
\end{aligned}$$

The growth-maximising inflation rate, therefore, can be determined as:

$$\overline{\pi} \in \left\{\pi \,\middle|\, \frac{\partial g}{\partial \phi}\cdot\frac{\partial \phi}{\partial \pi} = \left|\frac{\partial g}{\partial \pi}\right|\right\}$$

$$\begin{aligned}
\Leftrightarrow\quad & \frac{u^f}{\eta}\,\frac{p(\phi^*)\left(\left(1-u^f\right)r_K' - r_K u^{f\prime}\right)}{p(\phi^*)\left(p(\phi^*) + (1-u^f)\,r_K' - (r_K+\pi)\,u^{f\prime}\right) - p'(\phi^*)\left(\delta + r_K(1-u^f) - \pi u^f\right)} = \frac{u^f}{\eta} \\
\Leftrightarrow\quad & \frac{p(\phi^*)\left(\left(1-u^f\right)r_K' - r_K u^{f\prime}\right)}{p(\phi^*)\left(p(\phi^*) + (1-u^f)\,r_K' - (r_K+\pi)\,u^{f\prime}\right) - p'(\phi^*)\left(\delta + r_K(1-u^f) - \pi u^f\right)} = 1 \\
\Leftrightarrow\quad & p(\phi^*)^2 = p'(\phi^*)\left(\delta + r_K\left(1-u^f\right) - \pi u^f\right) \\
\Leftrightarrow\quad & \overline{\pi} = \frac{p(\phi^*)^2 - p'(\phi^*)\left(\delta + r_K\left(1-u^f\right)\right)}{-p'(\phi^*)\,u^f} > 0.
\end{aligned}$$

The impact on employment can be determined in a straightforward way by totally differentiating (6.16). ■

Similar to the spirit of the original Tobin model, therefore, we get an impact of portfolio choices on growth that is determined by the inflation rate (Walsh, 2001, ch. 2). However, due to the still negative impact of inflation on the overall return of the households' portfolio, there will only be a certain range of inflation rates for which the impact is positive, hence to the extent that monetary authorities control the inflation rate directly, there will be a growth maximising money supply rule. This, in turn, is in line with recent research on search models focussing on labour market search only and introducing money demand through a cash-in-advance rule (Wang and Xie, 2003).

**Corollary 4** *A central bank that only maximises a weighted sum of growth and inflation will not opt for the growth-maximising inflation rate but a lower one. A central bank that targets both growth and employment (besides inflation) runs the risk of an indeterminate optimal inflation rate if $\eta > 1$, with one of the equilibria being characterised by an inflationary bias.*

As already noted before for empirically plausible values of the intertemporal substitution elasticity, growth will reduce employment in this model (a result noted earlier by Eriksson, 1997) as a higher interest rate increases the opportunity costs of opening a vacancy. Hence growth and employment react in opposite ways to an increase in inflation, making the central bank (possibly) having an inflationary biais should it also target employment (in addition to output/productivity growth).

### 6.4.3 The government

The tax structure has in our set-up a particular role to play as taxes on labour and taxes on capital do not work the same way: Indeed, as can be seen from (6.15), only capital taxes will affect the equilibrium growth rates. Labour income taxes, on the other hand, will not impact on capital accumulation due to the triangular system of the financial and labour market interactions (financial market liquidity effects the labour market equilibrium but not vice versa) and will only affect the size of the economy through the labour supply effect.

Similarly to the impact of structural policies, the impact of taxes can be assessed using the full system of financial and labour market equations. While labour market liquidity reacts ambiguously with an increase in $\tau_K$ - it increases for $\eta > 1$ and decreases otherwise - financial market tightness, $\phi$, increases and so do interest rates. This, however, is not sufficient to soften the first-order impact of taxes on growth with unambiguously declines. In order to assess the impact of fiscal policy in isolation, we abstract from monetary policy, setting the inflation rate to zero, i.e. $\pi = 0$.

#### The Ricardian budget regime

In order to assess fully the impact of government activity, however, taxes cannot be considered in isolation but must be part of the overall budget constraint which - in the Ricardian case, i.e. in the absence of fiscal dominance - writes as:

$$G + r_B B = T + \dot{B}$$

where $T = \tau_K r_K K + \tau_w w N \equiv \tau Y$.

With government spending assumed to be constant ($G = 0$), any increase in government debt leads to higher interest payments that have to be matched by increases in tax rates, i.e. in a Ricardian fiscal regime, any increases in the debt service cannot simply be financed through higher deficits. Moreover, increases in the government deficit will increase financial market tightness as can be easily seen from the above national accounts: government bonds compete with bonds issued by firms, i.e. *ceteris paribus* an increase in government financing through bonds will raise $\phi$. The optimal tax policy can hence be analysed, as $\phi$ raises over and above the increase due to increased taxes. In this regard, a comparison of a no-tax regime ($\tau = 0, \phi^*$) with a regime with positive capital income taxes and positive government debt ($\tau > 0, \phi^{**}$), where $\phi^{**} > \phi^*$ due to government bonds, yields the following proposition:

**Proposition 5** *For sufficiently low equilibrium financial market tightness $\phi \leq \overline{\phi}$, there exists an interior value for tax-backed bond financing of government spending (i.e. Ricardian fiscal policy). The upper bound of a positive optimal tax policy depends on the share of government revenues financed through capital taxation, $\varpi = \frac{\tau_K}{\tau}$, i.e. $\overline{\phi} = \overline{\phi}(\varpi)$, $\overline{\phi}' < 0$.*

**Proof.** In the following we adopt the convention that $\phi(\tau_K) = \phi + \tau$ for convenience only. The financial market equilibrium is unaffected by capital taxes (only indirectly through impact on savings and consumption decision). The negative direct impact of taxes on growth increases with the equilibrium value of $\phi$:

$$\frac{\partial g}{\partial \tau_K} = -\frac{1}{\eta} r_K \left(1 - u^f\right) \leq 0, \; \frac{\partial^2 g}{\partial \tau_K \partial \phi} = -\frac{1}{\eta} \left[ \frac{\partial r_K}{\partial \phi} \left(1 - u^f\right) - r_K \frac{\partial u^f}{\partial \phi} \right] < 0$$

This has to be weighted against the positive effect of government bonds on the financial market equilibrium, which increases the growth rate:

$$\begin{aligned}
g &= \frac{\gamma(1 - \tau_K)\left(1 - u^f\right)}{\eta} r_K(\phi) - \frac{\rho + \pi u^f}{\eta} \\
\frac{\partial g}{\partial \phi} &= \frac{1 - \tau_K}{\eta} \left[ \left(1 - u^f\right) \frac{\partial r_K}{\partial \phi} - r_K \frac{\partial u_f}{\partial \phi} \right] - \frac{\pi}{\eta} \frac{\partial u_f}{\partial \phi} \\
\frac{\partial^2 g}{\partial \phi^2} &= \frac{1 - \tau_K}{\eta} \left[ -2 \frac{\partial u_f}{\partial \phi} \frac{\partial r_K}{\partial \phi} - r_K \frac{\partial^2 u_f}{\partial \phi^2} \right] - \frac{\pi}{\eta} \frac{\partial^2 u_f}{\partial \phi^2}
\end{aligned}$$

given that $\frac{\partial^2 r_K}{\partial \phi^2} = 0$. Under our functional assumption regarding the constant-elasticity-to-scale property of the matching process, $\frac{\partial^2 g}{\partial \phi^2}$ will be negative except for very small values of $\phi < \underline{\phi}$ with $\underline{\phi} > 0$. Moreover, we have that at $\phi = 0$:

$$\frac{\partial g}{\partial \tau_K} = 0, \; \frac{\partial g}{\partial \phi} \to \infty$$

Hence, a zero-tax policy cannot be optimal at $\phi = 0$. Given that $\left|\frac{\partial g}{\partial \tau_K}\right|$ is monotonically increasing with $\phi$ and $\frac{\partial g}{\partial \phi}$ is monotonically decreasing with $\phi$, at least from $\phi > \underline{\phi}$ onwards. Hence there will be a $\overline{\phi} > \underline{\phi} > 0$ such that $\frac{\partial g}{\partial \phi} + \frac{\partial g}{\partial \tau_K} = 0$ for $\tau_K > 0$.

When only $\varpi$% of public spending is financed through corporate taxation, then the growth rate writes as:

$$g = \frac{\gamma(1 - \varpi \cdot \tau)\left(1 - u^f\right)}{\eta} r_K(\phi) - \frac{\rho + \pi u^f}{\eta}$$

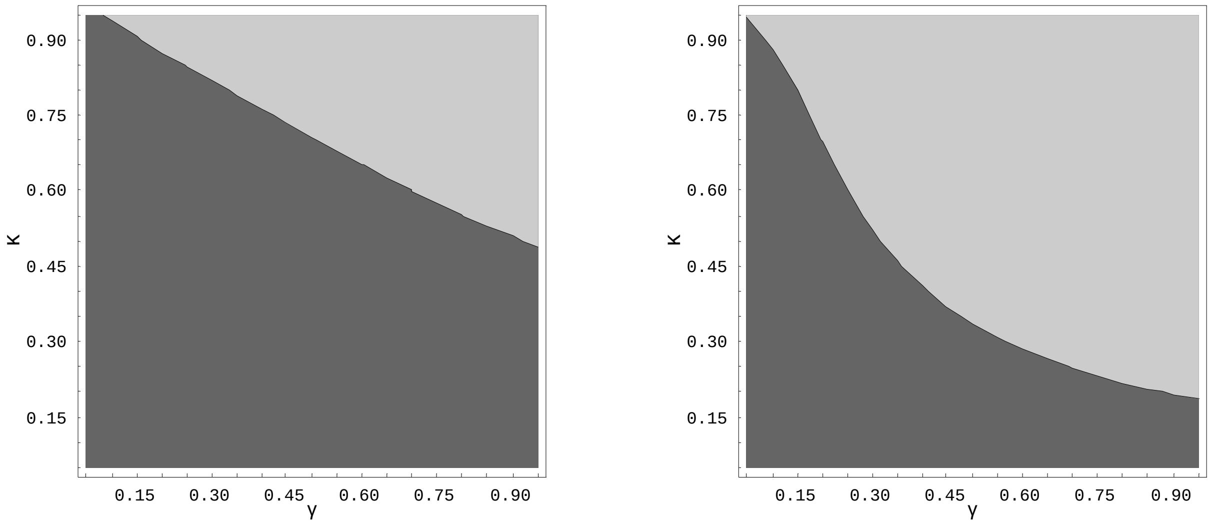
and consequently, the direct negative effect of a tax-financed increase in public debt is reduced:

$$\frac{\partial g}{\partial \tau} = -\varpi \frac{1}{\eta} r_K \left(1 - u^f\right) \leq 0, \; \frac{\partial^2 g}{\partial \tau \partial \varpi} = -\frac{1}{\eta} r_K \left(1 - u^f\right) \leq 0$$

hence $\frac{\partial g}{\partial \phi} + \frac{\partial g}{\partial \tau} (\varpi) = 0$ is satisfied for a financial market equilibrium $\overline{\overline{\phi}} (\varpi) > \overline{\phi} > 0$. ■

By introducing a competing asset that raises steady state interest rates, the government can affect the equilibrium growth rate above the rate obtained in a no-government equilibrium. This, however, is only the case for relatively low financial market tightness: the higher it is, the less of an impact will the introduction of government bonds on interest rates have. Unfortunately, there is no guarantee that the optimal tax policy will be positive at the financial market equilibrium defined by (6.17), and no closed-form algebraic solution can be derived. In order to get a sense of the equilibrium values, the following two graphs give an idea on the parameter space $(\gamma, \kappa)$ that would allow positive optimal tax policies, on for a fully capital-taxed financed budget (right panel) and one for a budget financed at one quarter by capital taxation, as is common in OECD countries.

Figure 6.1: Allowable parameter space for positive optimal tax rates.




(a) Full capital taxation financed public deficit ($\varpi$=1) (b) Public deficit financed through capital taxation by 25% ($\varpi$=0.25)

Note: The light grey shaded areas indicate the parameter tupels $(\kappa, \gamma)$ for which the financial market equilibrium is consistent with a positive optimal tax rate.

### 6.4.4 Interaction between monetary and fiscal policy

This section introduces coordination between monetary and fiscal policy in order to maximise the growth rate. In this case, we have to introduce seignorage into the government's budget constraint to account for the impact of different levels of inflation (and money supply growth). The wew government budget constraint including seignorage $(s)$ now writes as:

$$\dot{B} = r_B B - T + G - s$$

**Lemma 6** *Raising the money supply growth rate increases the optimal rate of taxation* $(\tau_K^*, \tau_w^*)$.

**Proof.** The inflation rate does not affect the direct negative impact of corporate taxation on growth. However, it has an impact on the way financial market liquidity affects growth. Recall that the growth rate increases with financial market liquidity at rate:

$$\frac{\partial g}{\partial \phi} = \frac{1-\tau_K}{\eta}\left[\left(1-u^f\right)\frac{\partial r_K}{\partial \phi} - r_K \frac{\partial u_f}{\partial \phi}\right] - \frac{\pi}{\eta}\frac{\partial u_f}{\partial \phi}$$

This derivative depends positively on the inflation rate:

$$\begin{aligned} \frac{\partial^2 g}{\partial \phi \partial \pi} &= \frac{\partial^2 g}{\partial \phi^2} \cdot \frac{\partial \phi}{\partial \pi} - \frac{1}{\eta}\frac{\partial u_f}{\partial \phi} \\ &= \frac{(1-\tau_K)\left(1-u^f\right) u^f \frac{\partial r_K}{\partial \phi} - ((1-\tau_K) r_K - \rho) \frac{\partial u_f}{\partial \phi}}{\left(\rho - (1-\tau_K)(1-u^f) r_K + \pi \cdot u^f\right)^2} > 0 \end{aligned}$$

which is unambiguously positive for non-negative growth rates (for which $r_K(1-\tau_K) > r_K(1-\tau_K)\left(1-u^f\right) > \rho$ holds). ■

**Proposition 7** *The growth rate can be maximised when government spending is financed through a combination of seignorage and bonds. The optimal growth rate of money is in this case lower than in the absence of government spending (and hence the inflation rate lower).*

**Proof.** This follows directly from the lemma above. ■

# 6.5 Conclusion

The preceding chapter has introduced financial and labour market search friction in an otherwise standard competitive dynamic general equilibrium model. The chapter has derived the stationnary state in the absence of growth and the balanced growth paths of both exogenous and endogenous growth.

The comparative statics around the steady state has revealed interesting properties of the model, in particular in the presence of (AK-)endogenous growth. In particular, the paper demonstrates the existence of an growth-maximising inflation rate, a (possible) trade-off between growth and employment maximisation, and the possibility for fiscal policies to increase the growth rate in case of very liquid financial markets (from the point of view of firms) by offering tax-backed public debt. Finally, positive interaction between monetary and fiscal policy has been shown to exist due to their complementary effect on the financial market equilibrium.

[The paper constitutes a first draft of the importance of financial market frictions in the search and matching variant for the proper assessment of monetary and fiscal policies. In particular, the effects of the macroeconomic policy mix is shown to exist even in the absence of price rigidities, as the model approach here focuses on the medium- to long-run. Further developments along these lines are in preparation and aim at the comparison of the effects described here with different enodgenous growth mechanisms (most notably the Schumpeterian one) and the development of a short-run model (with stick prices) that can be used to estimate and evaluate the effects of monetary and fiscal policies in OECD countries.]